The Pooling and Tranching of Securities: A Model of Informed Intermediation

Peter M. DeMarzo
Stanford University

I show that when an issuer has superior information about the value of its assets, it is better off selling assets separately rather than as a pool due to the *information destruction effect* of pooling. If, however, the issuer can create a derivative security that is collateralized by the assets, pooling and "tranching" may be optimal. If the residual risk of each asset is not highly correlated, tranching allows the issuer to exploit the *risk diversification effect* of pooling to create a low-risk and highly liquid security. In contrast, for an uninformed seller, pure pooling reduces underpricing and is preferred to separate asset sales. These results lead to a dynamic model of financial intermediation: originators sell pools of assets, some of which are purchased by informed intermediaries who then further pool and tranche them. Pooling and tranching allow intermediaries to leverage their capital more efficiently, enhancing the returns to their private information.

The repackaging of assets is ubiquitous in financial markets. For example, mortgage-backed securities (MBSs) are created by pooling a large number of individual home mortgages into a single financial trust. The trust is then sold to investors by selling separate classes, or tranches, of securities whose claims in aggregate represent a 100% interest in the trust, but which are individually highly heterogeneous. This securitization process has been applied to other assets, including car loans, credit card receivables, Western Union deposits, and commercial mortgages. A recent example is the collateralized bond obligation (CBO), created by pooling different junk-bond issues. The pool is then tranched into an investment grade "debt" security that ranks first in interest and principal payments, and a residual "equity" sliver in which the default risk is concentrated. Another innovation applies the concept recursively: the "kitchen-sink bond" is formed by tranching a low-risk debt security from a pool of residual pieces from other asset-backed securities.[1]

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[1] At a more abstract level, corporate securities can be viewed as a repackaging of the underlying human and physical assets of the firm into debt, equity, and other securities. Thus, while the analysis here focuses on the application to asset-backed securities, many of the results may also apply to corporate finance.

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These examples share the common feature of an issuer or financial intermediary pooling assets and then reselling the pool as a collection of new securities. With perfect capital markets, such repackaging would be irrelevant. But this is at odds with the reality of the accelerated growth of the asset-backed securities market and the substantial profits of the intermediaries involved. The goal of this article is to develop a rational equilibrium model of this process that is consistent with these and other stylized facts.

To explain the gains from repackaging securities, three market imperfections seem important: transactions costs, market incompleteness, and asymmetric information. This article focuses on asymmetric information. This does not deny the presence and importance of transactions costs or market incompleteness. Rather there are features of the market that seem best explained by asymmetric information. For example, market incompleteness cannot explain the construction of "pass-through" pools, which do not augment the span of tradeable claims. It is also unlikely to explain the CBO market, since good substitutes already exist for the debt and equity tranches that are created. Transactions costs imply an advantage to pooling but offer little help in rationalizing the pieces the pools are carved into. As we shall see, the asymmetric information model offers useful and consistent insights into these markets.

For some asset-backed securities, such as MBSs, many attributes of the underlying assets are public information. But information asymmetries still exist because the models used to price these securities are largely proprietary, and the value estimates produced by one's own model are an important piece of private information. For example, Bernardo and Cornell (1997) analyze MBS auctions and find that although all bidders were sophisticated investors or investment banks, their bids are extremely variable, with the winning bid exceeding the median bid by over 17% on average. They conclude that this variability is due to asymmetric information regarding valuation. Wallace (2001) provides additional evidence for this interpretation and documents the degree of heterogeneity across similar MBSs.

A goal of this article is to demonstrate the role of asset securitization in a model of informed financial intermediation. Consider a sophisticated financial intermediary that has superior ability in valuing assets. Based on this ability, the financial intermediary can profit by buying underpriced assets and holding them to maturity. However, to leverage its available capital fully, the intermediary would prefer to resell the assets (at their true value) and reinvest the proceeds (in newly identified underpriced assets). But given its superior information, the intermediary faces a "lemons" problem when it attempts to resell the assets, resulting in illiquidity: the price the intermediary receives for the assets is decreasing in the quantity sold. By pooling the assets and issuing a high quality (and information

insensitive) security tranche backed by the pool, the intermediary can mitigate the lemons problem and maximize its return on capital.

To build this model of financial intermediation, I begin by examining the consequences of pooling assets for an informed issuer. The analysis builds on the security design model of DeMarzo and Duffie (1999) (henceforth "D&D") and the signaling model of Leland and Pyle (1977). These articles develop models in which the seller signals a high value security by its willingness to retain a portion of the issue. In Section 3, I show that pooling of assets prior to sale is not advantageous to an informed issuer. Pooling assets destroys the asset-specific information held by the issuer, eliminating its option regarding how aggressively to sell each asset. This *information destruction effect* reduces the issuer's payoff.

In Section 4, I consider the effect of pooling and tranching. Building on results from D&D and DeMarzo (2003b), I show that there is a beneficial *risk diversification effect* of pooling, allowing the issuance of a low-risk debt security from a large pool. This low-risk debt is less sensitive to the issuer's private information, and hence is more liquid. I show that as the size of the pool grows large, the risk diversification effect dominates the information destruction effect, so that pooling and tranching is optimal for an informed issuer.

Having analyzed the case of an informed issuer, in Section 5 I consider the problem faced by an uninformed issuer when some buyers are potentially informed. For example, many assets are created by "originators" who specialize in marketing and other customer services. These assets may then be sold to uninformed investors or acquired by an intermediary who is more sophisticated and more informed about asset values. Because uninformed investors face an adverse selection problem—the informed intermediary buys those assets it knows to be of high quality—originators are forced to underprice assets, as in Rock's (1986) model of IPO underpricing. Here I show that an uninformed issuer does have an incentive to pool the assets even if they are not tranched prior to sale. Pooling prevents the informed investors from selectively purchasing just the highest quality components of the pool, reducing the adverse selection problem facing uninformed investors.

While useful on their own, combining these results leads to a model of informed intermediation, developed in Section 6. Uninformed originators pool assets to reduce underpricing. Informed intermediaries purchase the highest quality pools from originators. Then, to raise capital for future purchases, intermediaries further pool the asset pools and issue low-information-sensitive security tranches backed by these large pools. The ability to repackage securities enhances the returns to information.

This stylized model fits well the market for many asset-backed securities. For example, mortgage originators generally pool the mortgages they originate into a pass-through MBS consisting of 20–30 mortgages. Since

1995, over 50% of all mortgages that originated in the United States have ultimately been pooled in this fashion.[2] These pass-throughs are sold to intermediaries who then combine 100–300 of these MBS pools into a real estate mortgage investment conduit (REMIC). The intermediaries issue securities backed by the REMIC known as collateralized mortgage obligations (CMOs). The most liquid of these CMOs are generally designed to be relatively insensitive to the rate of mortgage prepayment, consistent with the notion that the intermediaries themselves are likely to be best able to evaluate and price prepayment risk.[3] Consistent with the model, while intermediaries sell off many of these CMOs, they also retain significant fractions for their own portfolios [see Wallace (2001) for a detailed analysis of this market and its participants].

1. Related Literature

Leland and Pyle (1977) develop a signaling model of liquidity in which a risk-averse entrepreneur can diversify by selling an equity stake in his firm. In equilibrium, the entrepreneur signals his private information about firm value by retaining equity, and the market's demand curve for equity is (rationally) downward sloping. Leland and Pyle conjecture that signaling costs might be reduced by combining many projects, leading to specialization in information production. Rather than risk aversion, the motive for trade in this article is that the issuer has an above market discount rate due to the availability of other positive net present value (NPV) investments. By similar reasoning, an owner signals an asset's high quality by retaining some of it. In either case, however, the Leland and Pyle conjecture fails in that simply pooling assets will not reduce asymmetric information costs and enhance liquidity.[4]

Diamond (1984) analyzes financial intermediation based on an ex post asymmetry of information — investors do not observe firms' cash flows. To ensure repayment, investors can undertake costly monitoring of the firm. With multiple investors, monitoring entails duplicative effort that can be avoided by delegating monitoring to a financial intermediary. Moreover, as the intermediary grows large (by pooling independent securities), the intermediary can offer investors a nearly risk-free debt contract. Similarly, Diamond (1993) and Winton (1995) argue that the issuance of

[2] Data from The Bond Market Association (www.bondmarkets.com).

[3] Prepayment risk is the most important risk for MBSs. Credit risk is not an issue due to the guarantees typically provided by various agencies (Fannie Mae, Freddie Mac, Ginnie Mae). For other asset-backed securities, credit risk is generally quite important. Consistent with the model, for these asset-backed securities, pools are generally tranched into prioritized securities, with the most senior securities being relatively insensitive to information regarding credit quality.

[4] While this article focuses on a risk-neutral issuer, I extend the model to the case of a risk-averse issuer in the working paper version [DeMarzo (2003a)].

different classes of securities, varying in seniority, can reduce monitoring costs. Like these analyses, I derive debt as an optimal contract for a large intermediary. On the other hand, I focus on an ex ante information asymmetry in which sophisticated investors become intermediaries because they have superior information about asset values. This type of information is probably more relevant for many asset-backed securities. For example, monitoring the cash flows of CMOs is not typically a problem, whereas there is general agreement that the major investment banks have a superior ability to value them.[5]

The Gorton and Pennachi (1990) model of intermediation features two investor clienteles, informed and uninformed. Informed investors exploit the uninformed investors when the uninformed are in need of liquidity. In their model, it is optimal for the uninformed to form an intermediary that splits cash flows into riskless debt and equity. The uninformed can then trade the debt claims to satisfy their liquidity needs, avoiding losses from trading with the informed. In contrast with our model, the intermediary in the Gorton and Pennachi model is uninformed.

Winton (2001) considers a model in which an intermediary (a bank) has an incentive to monitor and acquire information about the underlying assets (a firm) in order to reduce agency costs. The intermediary may suffer a liquidity shock, however, and be forced to issue claims backed by its holdings. Similar to D&D, he shows that as an informed seller, the intermediary's liquidity costs are reduced if it holds debt rather than equity. Winton does not consider the possibility of pooling and tranching the securities of multiple firms.[6]

The Glaeser and Kallal (1997) model of asset-backed securities incorporates an issuer's choice of whether to gather information. They note that pooling assets has ambiguous effects on an issuer's incentives to become informed and therefore on the liquidity of the pool. The model here extends their analysis by allowing the issue of derivative tranches in addition to simple pass-through securities, which I demonstrate to be critical. Riddiough (1997) also examines security design for asset-backed securities. He notes that splitting off a riskless security is beneficial since the issuer will suffer no asymmetric information losses on that security. The analysis here is more general and does not require that the security tranche be riskless. Riddiough focuses on the agency and governance issues that arise in this setting, issues not addressed here.

Subrahmanyam (1991) and Gorton and Pennachi (1993) explore how pooling equity securities can reduce adverse selection when uninformed

[5] Ex post monitoring may be a more important issue for collateralized loan obligations.

[6] In fact, the pooling and tranching of bank loans into collateralized loan obligations is a rapidly growing class of asset-backed securities.

liquidity traders trade with informed traders. Axelson (1999) explores this issue in an auction context in which buyers have differential information, showing that as the number of assets grows large, auction revenues can be improved by pooling assets prior to sale. His analysis corresponds most closely to that of Section 5, where I also demonstrate that pooling reduces the adverse selection problem.

2. The Underlying Assets

Consider the problem faced by an issuer who holds n assets, but prefers to hold cash. This issuer must choose whether to sell the assets separately, as a pool, or as an asset-backed security based on the pool. To evaluate the issuer's acquisition and sale decisions, introduce the following assumptions regarding the underlying assets.

Each asset i has a final nonnegative payoff of $Y_i = X_i + Z_i$. The component X_i represents the private information of the issuer and Z_i is the remaining risk the issuer faces. Let $Y \equiv (Y_1, \ldots, Y_n)$ denote the vector of payoffs and $Y^n \equiv \sum_{i=1}^{n} Y_i$ denote the cumulative payoff of the assets, and similarly for X and X^n as well as Z and Z^n. Finally, introduce the notation $X_{-i} \equiv (X_1, \ldots, X_{i-1}, X_{i+1}, \ldots, X_n)$.

The first assumption is without loss of generality:

1. $E[Z_i \mid X] = 0$; or equivalently, $X_i = E[Y_i \mid X]$.

This assumption states that X_i embodies all of the information known to the issuer regarding the expectation of the cash flow Y_i. (Equivalently, we could let I represent the issuer's information and then define $X_i \equiv E[Y_i \mid I]$.)

Next, introduce the technical assumption:

2. Given any X_{-i}, the conditional support of X_i is a closed interval.

This assumption implies that whatever information the issuer has revealed about the assets other than i, there still remains a continuum of possible information states regarding asset i.

The last assumption is substantive, but only mildly restrictive:

3. Given any X_{-i}, the conditional support of X_i has greatest lower bound $x_{i0} > 0$.

That is, the "worst-case" outcome of X_i is independent of X_{-i}. Given this assumption, the support of X^n is an interval with greatest lower bound $x_0^n = \Sigma_i x_{i0}$. While this last assumption implies that any two assets X_i and X_j are not perfectly correlated, it does not rule out any other correlation. For example, if the assets represent loans, knowing the quality of loan i might affect the probability that loan j is "bad." The assumption simply states that this probability does not go to zero — whatever the quality of other loans, it remains *possible* that loan j is bad.[7]

Each asset is perfectly divisible. Owning a fraction $q_i \in [0, 1]$ of asset i entitles the owner to the cash flows $q_i Y_i$. In the various applications in the rest of this article, I investigate the relationship between the market price for the assets and the fraction sold by the issuer when there is illiquidity resulting from asymmetric information.

Finally, assume there exists a large number of risk neutral investors. For convenience, also assume that the market interest rate is zero. Hence, in the absence of asymmetric information (i.e., if X were public), each asset could be sold for a market price of X_i.

3. Pooling and Information Destruction

In this section I consider an informed issuer who sells assets either individually or as a pool. I show that the issuer's payoff exhibits a natural convexity resulting from the issuer's option to choose the quantity of each asset to sell based on its private information. Pooling destroys this option and raises the costs the issuer must bear to signal the quality of the assets. This information destruction effect implies that it is not optimal for the issuer to sell the assets as a single pool. Rather, the issuer's payoff is highest if each asset is sold individually to the market.

I model a risk-neutral issuer who discounts future cash flows at a higher rate than other investors.[8] Thus, the issuer would prefer to sell the assets for cash. This corresponds to the model of D&D and can be motivated by supposing that the issuer has access to other investment opportunities with an above market return. In particular, if the issuer earns a profit buying and selling assets, the issuer may wish to raise cash to fund new asset purchases. In Section 6, I model this process and endogenously determine this preference for cash.

Thus, suppose the issuer is risk neutral and has a discount factor $\delta < 1$, and suppose the issuer sells a fraction q of the entire pool of assets to investors at a market price for the pool of p. Then the payoff to the issuer is given by

$$E[\delta(1-q)Y^n + qp \mid X] = \delta X^n + q(p - \delta X^n).$$

[7] The positivity of x_{i0} does *not* imply strictly positive cash flows Y_i, but only requires that the security retains some positive "option" value even given the worst-case information.

[8] An alternative motive for trade is risk-sharing, as in Leland and Pyle (1977). The intuitions of this model are robust to that setting as well, as I show in the working paper version of this article [DeMarzo (2003a)].

If the issuer anticipates a market demand schedule given by $P(q)$ for the pool, then given $X^n = x$ the issuer will issue quantity $Q^P(x)$ that solves

$$\max_q \delta x + q(P(q) - \delta x) = \delta x + \max_q q(P(q) - \delta x). \tag{1}$$

That is, the total payoff to the issuer is the issuer's discounted value of the asset plus any "profit" from the sale of the security. The issuer chooses a quantity to sell to maximize this profit, which we denote by

$$\Pi^P(x) = \max_{q \in [0,1]} q(P(q) - \delta x). \tag{2}$$

The following key properties follow immediately from the definition of Π^P:

Lemma 1. *For any demand function P, the issuer's profit Π^P is decreasing*[9] *and convex in x. Also, the fraction $Q^P(x)$ that the issuer sells is decreasing in x.*

Proof. For fixed q, the issuer's objective is decreasing and linear in x with slope $-q\delta$. Hence Π^P is the upper envelope of linear functions and is therefore convex. The fact that q is decreasing follows from the convexity of Π^P and the fact that it has $-q\delta$ as a subgradient. ■

The properties described in Lemma 1 are clearly robust and will drive much of the subsequent analysis. However, it is useful to describe an equilibrium in which investors' demand is not arbitrary, but is based on their perceived value of the pool given the issuer's decision. In a standard rational expectations or Bayes–Nash equilibrium, investors' demand satisfies

$$P(Q^P(X^n)) = E[X^n | Q^P(X^n)]. \tag{3}$$

Note that from Lemma 1, for any such equilibrium the demand schedule P is (weakly) downward sloping in the range of Q. We say that the equilibrium is *separating* if $P(Q(X^n)) = X^n$. The following characterization of the equilibrium is from D&D:[10]

Lemma 2. *Given the worst-case asset value $x_0 > 0$, there is a unique separating equilibrium, given by $Q^*(x) = (x/x_0)^{\frac{-1}{1-\delta}}$ and $P^*(q) = x_0 q^{\delta - 1}$. The equilibrium payoff function $\Pi^*(x) = \pi(x/x_0)x_0$, where $\pi(x/x_0) = (1-\delta)(x/x_0)^{\frac{-\delta}{1-\delta}}$.*

[9] Here and throughout I use the term decreasing/increasing in the weak sense (as opposed to *strictly* decreasing/increasing).

[10] As with all signaling models, multiple equilibria are possible in the absence of restrictions on out-of-equilibrium beliefs. For the model considered here, this equilibrium is the unique equilibrium satisfying standard refinements (see D&D).

Proof. See D&D. The solution follows from differentiation of Equations (2) and (3) and the boundary condition $Q^*(x_0) = 1$. ■

Note that the issuer's quantity choice depends upon the ratio of the asset value to its worst-case value x/x_0.[11] Thus the issuer's equilibrium payoff is homogeneous of degree 1 in x and x_0. While having an explicit functional form for π is convenient, recall that Lemma 1 implies that π is decreasing and convex without further calculation.

Next, suppose the issuer sells the assets separately. Consider the sale of asset i. Essentially, the issuer faces the same problem as in Equation (2) above with X_i in place of X^n. The investors are also in an analogous position, with the possible exception that they may have learned information about X_j from the prior sale of asset $j \neq i$ that might alter their conditional distribution for X_i. However, the equilibrium depends only on the worst-case outcome of the expected asset payoff and not on the distribution itself. By our initial assumptions, the worst case is not affected by X_{-i}. Hence the equilibrium is unchanged. This leads to the following:[12]

Lemma 3. *If the issuer sells assets separately, there is a unique separating equilibrium in which the issuer's total payoff is given by $\sum_{i=1}^n \pi(X_i/x_{i0})x_{i0}$.*

Thus, the issuer's payoff from a separate or a pooled sale of the assets can be compared, yielding the main result of this section:

Theorem 1. *The issuer prefers a separate sale of the assets to a pooled sale; that is,*

$$\sum_{i=1}^n \pi\left(\frac{X_i}{x_{i0}}\right) x_{i0} \geq \pi\left(\frac{X^n}{x_0^n}\right) x_0^n.$$

where the inequality is strict if X_i/x_{i0} is not equal for all i.

Proof. By the convexity of π, and the fact that $x_0^n = \sum_i x_{i0}$, we have

$$\sum_{i=1}^n \frac{x_{i0}}{x_0^n} \pi\left(\frac{X_i}{x_{i0}}\right) \geq \pi\left(\sum_{i=1}^n \frac{x_{i0}}{x_0^n} \frac{X_i}{x_{i0}}\right) = \pi\left(\frac{X^n}{x_0^n}\right).$$

The strict inequality follows from the fact that Q^* is strictly decreasing so that π is strictly convex. ■

Theorem 1 demonstrates that an informed seller will prefer to sell securities individually rather than as a single pass-through pool of

[11] As in all separating equilibria, the equilibrium is sensitive to the support assumption, x_0, and otherwise insensitive to the distribution of X, which may seem unnatural. In Section 6 of the article, I show, in a dynamic setting, how x_0 arises endogenously based on the full distribution of X.

[12] See appendix for proofs not in the text.

securities. Intuitively, because the issuer holds an option regarding the quantity of the asset to sell, the issuer's payoff is convex in the privately observed quality of the asset. Therefore, the issuer prefers not to combine high and low quality assets to create a medium quality pool. I refer to this loss as the information destruction effect of pooling.

The information destruction effect follows from the convexity of π and not its explicit functional form. Thus, the result is more general than the explicit setting considered here. For example, it extends to the Leland and Pyle (1977) model, in which the issuer is risk-averse (see DeMarzo (2003a)).

Theorem 1 relies on the assumption that $x_0^{\Pi} = \sum_i x_{i0}$; that is, the worst possible pool is equal to a pool of the worst possible assets. There may be cases for which this does not hold. For example, investors may have data regarding characteristics of the pool that improves the worst-case scenario, so that $x_0^{\Pi} > \sum_i x_{i0}$. In this case there may be benefits associated with pooling even for an informed issuer.[13]

4. Tranching and Risk Diversification

In the previous section, the issuer could either issue the assets separately or as a pool, and pure pooling was shown to be suboptimal. In this section, I allow the issuer to create a derivative security based on the cash flows of the underlying asset or pool of assets. I then show that pooling the assets and selling a derivative tranche is superior to both pure pooling and separate asset sales.

Consider an issuer with assets with payoff $Y = X + Z$. Rather than sell shares in asset i directly, the issuer may create a security or tranche that pays $F(Y_i)$ for some measurable function F. Restricting attention to limited-liability securities that are backed solely by the underlying assets implies $F(y) \in [0, y]$. Such a security F is referred to as an "asset-backed security." For tractability, I consider only nondecreasing functions F.[14] Since the goal is to show that pooling and tranching is superior to both individual sales and pure pooling, restricting attention to monotone tranches only strengthens the result.

Given risk neutrality, with pure pooling the remaining risk Z plays no role. However, the creation of nonlinear securities F implies that Z plays a critical role—the risk inherent in Z will determine the extent to which a security can be designed that minimizes the asymmetric information between the issuer and investors. I show that the risk diversification effect of pooling is beneficial in this regard. In fact, it can overcome the information destruction effect, so that pooling and tranching is optimal for the issuer.

At the time of issue, X is private information of the issuer. But because there are usually significant delays between the design of a security and its sale, X may or may not be known at the time that the security design F is chosen. Therefore, I consider below both ex ante and ex post security design, and show that they lead to similar conclusions.

4.1 Ex ante security design

Suppose that the security design F is chosen prior to the realization of the information X. In this case, the issuer's choice of F does not reveal any information. This timing is relevant in several applications. First, asset-backed security designs may be standardized, and thus not reflective of private information relating to a particular issue. Second, there are often significant delays between the design of the security and its sale. If private information is acquired continuously, significant information may be learned during this delay.[15] Third, the informed issuer may be an underwriter who did not directly control the design.

To simplify notation, suppose for the moment that there is a single asset ($n = 1$) with payoff Y. Given a security design F and private information X, let the expected payoff of the security be given by $f \equiv E[F(Y)|X]$. Suppose the issuer sells a fraction q of the security F for a price p. Then the issuer holds qp in cash, and assets with discounted value $\delta(Y - qF(Y))$. The issuer's expected payoff is therefore

$$E[\delta(Y - qF(Y)) + qp \mid X] = \delta X + q(p - \delta f).$$

Suppose the issuer anticipates a market demand schedule given by $P^F(q)$ for the security F. Then given the conditional value f of the security, the issuer solves

$$\Pi^{P^F}(f) = \max_q q(P^F(q) - \delta f).$$

The structure of this problem is identical to Equation (2) of Section 3, and so we have the following characterization (see D&D):

Lemma 4. *Let $[f_0, f_1]$ be the support of $f = E[F(Y)|X]$. There is a unique separating equilibrium with equilibrium payoff function $\Pi^*(f) = \pi(f/f_0)f_0$, where π is defined in Lemma 2.*

[13] For example, if a credit card issuer has better information only regarding the *identity* of bad accounts, rather than their number, then pooling all accounts can improve the worst-case scenario.

[14] This restriction is for tractability. Since the goal is to show that pooling and tranching is superior to both individual sales and pure pooling, restricting attention to monotone tranches only strengthens the result.

[15] For example, if the issuer's information is the output of a proprietary valuation model, it is the model's valuation on the day of sale (based on the current yield curve, etc.) that is relevant.

This result gives the issuer's profit given a security design F and a conditional value of $f = E[F(Y)|X]$. Because X is not known at the time F is chosen, the issuer's ex ante expected profit given F is $E[\pi(f/f_0)f_0]$. Hence, the ex ante security design problem is the following:

$$G[Y] = \max_{F(\cdot)} E[\pi(f/f_0)f_0]. \tag{4}$$

Before proceeding, note the following properties of the ex ante payoff function G:[16]

Lemma 5. *G is homogeneous of degree 1; that is, $G[aY] = aG[Y]$. Also, $G[Y] \leq (1-\delta)x_0$, and the inequality is strict if X and Z are independent and continuously distributed.*

The upper bound in Lemma 5 states that the issuer can at best recover the "retention cost" $(1-\delta)$ on the worst-case value x_0. If a security design provided a higher payoff, it would be imitated by the worst type.

From Equation (4), the issuer must trade off, making the worst-case payoff f_0 of the security as high as possible, while at the same time minimizing its information sensitivity f/f_0 (since π is decreasing). While the set of possible security designs is vast, D&D show that for standard distributions, the trade-off is best accomplished with a standard debt contract, which pays the lowest, most information insensitive, cash flows first.

Lemma 6. *Suppose Z is independent of X and has a log-concave[17] density function. Then the optimal monotone security design is a standard debt contract. That is, $F^*(Y) = \min(d, Y)$ for some constant d.*

Proof. Given the additive separable construction of Y, the assumption on Z implies that the conditional distribution of Y given X satisfies the monotone likelihood ratio property (MLRP). This is stronger than the "uniform worst-case" condition of D&D. They demonstrate that this condition implies standard debt is the optimal monotone security design. ■

Thus it is sufficient to consider standard debt contracts, replacing Equation (4) with

$$G[Y] = \max_{d} E[\pi(f^d/f_0^d)f_0^d],$$

where $f^d = E[\min(d, Y)|X]$. Let $D^*[Y]$ represent the optimal face value of the debt.

[16] Note that G operates on the random variable Y, not its outcome, analogous to an expectation operator. I use square brackets, $[\cdot]$, to denote such operations.

[17] The density function g is log-concave if $\log(g(s))$ is concave in s. This property is satisfied by many standard distributions, such as uniform, normal (possibly truncated), and exponential (possibly truncated).

Now consider whether an issuer with multiple assets ($n > 1$) is better off selling a single debt security backed by the pooled assets, versus selling separate debt securities each backed by a single asset.[18] The comparison requires knowledge of how the asset risks are related. For simplicity, I assume a one-factor structure for the residual risk:

Assumption 1. *$Z_i = \varepsilon_i + \eta$, where the idiosyncratic risk ε_i is independent of $(\varepsilon_{-i}, \eta, X)$ and the common risk η is independent of (ε, X). Also, ε_i and η have log-concave density functions.*

Under this assumption, for a pool of size n, $Z^n = (\sum_i \varepsilon_i) + n\eta$. Since log-concavity is preserved by convolution [Prékopa (1973)], the conditions of Lemma 6 are satisfied. Hence, if the assets are pooled, the issuer's ex ante expected payoff is given by

$$G\left[\sum_{i=1}^{n} Y_i\right]. \tag{5}$$

If instead the issuer does not pool the assets, they can be sold individually. Rather than selling each asset outright, however, the issuer can construct a new security for each asset i which is backed by that asset. By an argument identical to that in the proof of Lemma 3, this problem is separable across assets, and the aggregate ex ante profits to the issuer from separate sales is given by

$$\sum_{i=1}^{n} G[Y_i]. \tag{6}$$

Therefore, the decision to pool or not amounts to a comparison of Equations (5) and (6). Using the homogeneity of G, the issuer prefers to pool the assets prior to tranching if it leads to a higher per-asset payoff:

$$G\left[\frac{1}{n}\sum_{i=1}^{n} Y_i\right] \geq \frac{1}{n}\sum_{i=1}^{n} G[Y_i].$$

Next I show the key result of this section: If the residual risk of the assets is diversifiable, then for large enough n, it is optimal for the issuer to pool the assets prior to tranching them. To state this result, we first suppose that the per-asset worst-case payoff is well defined in the limit, $\lim_{n\to\infty} \frac{1}{n}\sum_i x_{i0} = x_0$, and that the asset payoffs are nondegenerate in the sense of Lemma 5 so that there is some loss relative to the theoretical

[18] We could compare the payoff from pooling and tranching versus the payoff from simply selling the assets individually. Since issuing debt against an individual asset is superior to selling the asset outright, the comparison undertaken is a stricter test of the superiority of pooling.

maximum payoff of $(1-\delta)x_{i0}$ for each asset:

$$\overline{\lim_{i\to\infty}}\ \frac{G[Y_i]}{(1-\delta)x_{i0}} < 1. \tag{7}$$

Now we show that with pooling, when the residual risk is diversifiable the payoff per asset approaches the theoretical maximum of $(1-\delta)x_0$.

Theorem 2. *Suppose $\eta=0$ and that Y_i have bounded second moments. Then as $n\to\infty$,*

$$G\left[\frac{1}{n}\sum_{i=1}^{n} Y_i\right]\to(1-\delta)x_0 \quad \text{and} \quad D^*\left[\frac{1}{n}\sum_{i=1}^{n} Y_i\right]\to x_0.$$

Thus, pooling and tranching is optimal for sufficiently large n.

Contrast this result with Theorem 1 of Section 3, which showed that due to the information destruction effect, pure pooling reduces the issuer's profits. Theorem 2 shows that the issuer can benefit from pooling *and* tranching a large number of securities. The intuition for this result is a second effect associated with pooling, the risk diversification effect. Even though all agents are risk neutral, diversification is valuable because it allows the issuer to construct a low-risk security with greater liquidity. When the residual risk is diversifiable, the issuer can issue debt with a face value of x_0 that is nearly risk-free and thus insensitive to the issuer's private information. While diversification improves the issuer's payoff, note that the consequences of asymmetric information are still present in the limit—the issuer's payoff is bounded by $(1-\delta)x_0$, whereas in the first-best the issuer could recover $\lim_{n\to\infty}(1-\delta)\frac{1}{n}E[X^n]$.

To verify that the gain from pooling results from risk diversification, consider an alternative scenario in which the residual risks of the assets are perfectly correlated. In this case, there is no risk diversification from pooling, and so only the information destruction effect applies. This is confirmed by the following result:

Theorem 3. *Suppose $\varepsilon_i=0$ and $x_{i0}=x_0$ for all i. Then for any n, pooling is not optimal.*

Thus, the benefits from pooling depend on the degree of diversification that results. Since the cost of pooling is the information destruction effect, the gain from pooling depends on the nature of the information. In particular, if the information X_i is specific to each asset and thus independent across assets, information destruction will be more severe than if the information X_i is more general, and hence positively correlated across assets. That is, the payoff from pooling is increasing in the

riskiness of X^n:

Theorem 4. *Let $Y_i=X_i+\varepsilon_i+\eta$ and $\hat{Y}_i=\hat{X}_i+\varepsilon_i+\eta$, and suppose $x_{i0}=\hat{x}_{i0}$. If X^n is a mean-preserving spread of $\hat{X}^n$, then $G[\frac{1}{n}\sum_i Y_i]\ge G[\frac{1}{n}\sum_i \hat{Y}_i]$.*

Theorem 4 implies that the issuer is better off if the private information is general, rather than specific to each asset. For example,

Corollary. *Let $Y_i=X_i+\varepsilon_i+\eta$ and suppose the private information X_i is composed of J independent factors $X_i=\sum_{j=1}^{J}\xi_{ji}$. Suppose each factor j is either common, such that $\xi_{ji}\equiv\xi_j$ for all i, or unique, such that ξ_{ji} are independent draws with the same distribution as ξ_j for all i. Then the issuer's payoff from pooling and tranching is increasing in the number of common factors.*

Proof. Note that the distribution of each X_i is the same whether the factors are common or unique. Only the distribution of X^n changes. The result then follows from the observation that ξ_j is a mean-preserving spread of $n^{-1}\sum_i\xi_{ji}$. ■

Figure 1 illustrates a numerical example, plotting the expected payoff per asset if the issuer issues debt with face value d per asset that is backed by a pool of n assets. For $n=1$, the optimal face value of the debt for the

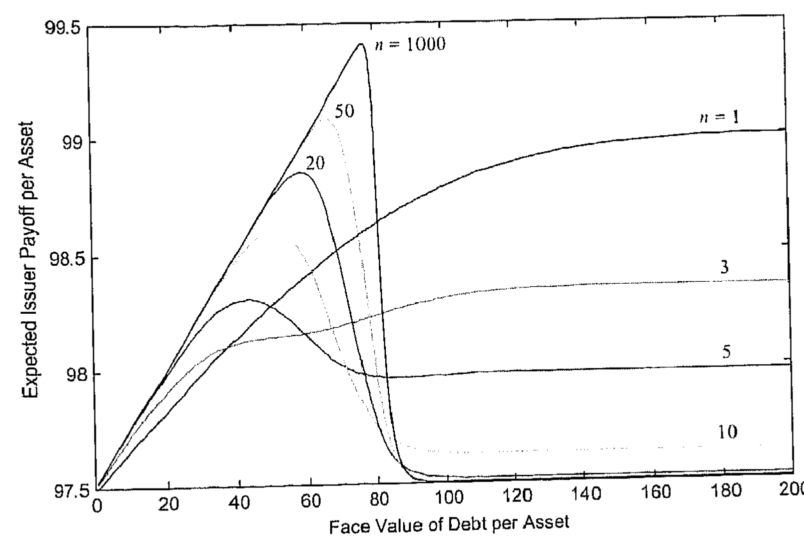

Figure 1
Per-asset payoff for different levels of debt and pool size
$\delta=.975$, $E[Y_i]=100$, X_i's are i.i.d. binomial on {80, 160} with probabilities {75%, 25%}, ε_i are i.i.d. normal with a volatility of 50, and $\eta=0$. (A binomial distribution for X_i is used for convenience; we can view it as a limiting case of a continuous support where the density on [80, 160] converges to zero.)

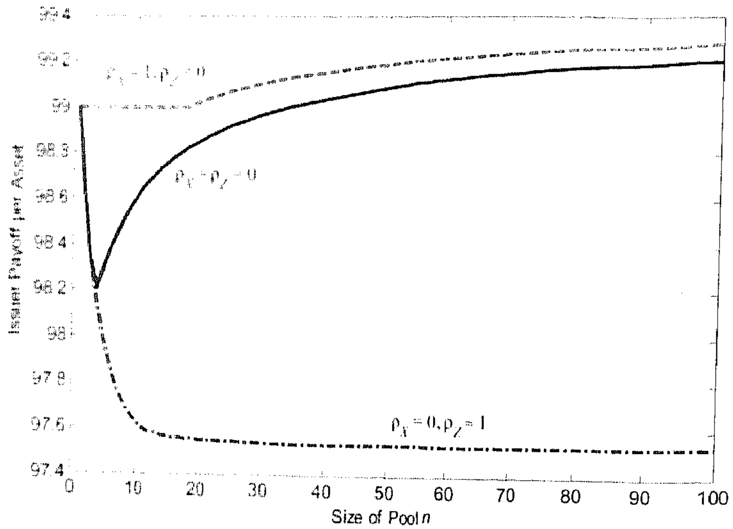

Figure 2
Per-asset payoff based on pool size for different correlations between information and residual risk (parameters match Figure 1)

issuer is $d = \infty$; that is, the issuer chooses $F(Y) = \min(d, Y) = Y$, a pure pass-through security.[19] As n increases, however, the payoff associated with the pure pass-through decreases, as implied by Theorem 1 of Section 3. For $n \geq 5$, it is optimal for the issuer to create a debt security that is backed by the pool. As n increases, the issuer's payoff increases due to the increased risk diversification. Ultimately, for n sufficiently large, pooling and tranching dominates individual asset sales.

Figure 2 illustrates the effect of correlation on the payoff to the issuer. This figure shows the payoff per asset as a function of the size of the pool, assuming the issuer constructs the optimal debt tranche. If Z_i approach perfect positive correlation, there is no diversification from pooling. Hence, by Theorem 3, pooling makes the issuer worse off due to the information destruction effect. On the other hand, if X_i approach perfect positive correlation, there is no information destruction due to pooling. Thus pooling benefits the issuer. Finally, for intermediate cases, both effects are operative. If the Z_i are uncorrelated, then pooling is optimal for n exceeding a threshold.

These results imply that the issuer will benefit most from pooling and tranching when the issuer's private information is general (the X_i are positively correlated) and the risks are specific (the Z_i are uncorrelated). This result may explain the desire for geographic diversification in mortgage pools, or industry diversification in collateralized bond obligations, while at the same time explaining the tendency not to combine underlying asset classes (e.g., mortgages and corporate bonds), since for these different asset classes the private information is likely to be uncorrelated.[20]

[19] While the graph only illustrates d up to 200, this is close to the limiting value for the given parameters.

4.2 Multiple tranches and ex post security design

Thus far, we have allowed the issuer to sell a single tranche for each asset pool. The tranche is designed prior to learning X, and the quantity to be sold is determined after X is known. However, the issuer may be able to do better by (i) using multiple tranches, and/or (ii) postponing the security design until after X is known. In this section I explore this possibility, and show that while the solution to the signaling equilibrium changes, the qualitative results of Section 4.1 continue to hold. In particular, the risk diversification benefit of pooling is still present, and leads to pooling and tranching being optimal given sufficient diversification.

If the issuer creates multiple tranches for an asset pool, then once the information X is learned the issuer will choose a quantity of each tranche, or a tranche portfolio, to sell to investors. This portfolio itself can be interpreted as a security design, in that its payoff is equivalent to some function F of the payoff of the asset pool. In a companion article [see DeMarzo (2003b)], I show that issuer's payoff is increasing in the number of tranches. Further, if the number of tranches is unlimited, and are restricted so that each tranche has a monotone payoff, the equilibrium is equivalent to a setting in which the security design is chosen ex post. Thus, I describe here the ex post security design problem,[21] but note that it is equivalent to the case of unlimited tranching.

Consider an issuer with a single asset with payoff $Y = X + Z$, and for simplicity maintain Assumption 1 so that X and Z are independent. Given the private information X, the asset has a private valuation of δX to the issuer. If, rather than hold the asset, the issuer designs and sells the asset-backed security F for price p, the issuer's payoff is given by[22]

$$E[\delta(Y - F(Y)) + p|X] = \delta X + (p - \delta E[F(Y)|X]).$$

That is, the issuer receives the private valuation δX plus the surplus generated by the sale of the security F.

[20] The argument here is most relevant when the issuer's information is related to variables common to all assets in a given class (such as risk premia), as opposed to cash flow data specific to an industry or locale.

[21] Nachman and Noe (1994) also consider the ex post design problem. However, in their model the issuer raises a fixed amount of capital, and so a pooling equilibrium results. Here the amount of cash raised is variable, allowing for separation based on the security issued.

[22] In this case, there is no need to separate the quantity decision from the design decision, since selling fraction q of security F is equivalent to selling all of the security qF.

The issuer chooses the security design F taking as given the market demand function for securities, given by some function P such that $P[F]$ is the price that investors will pay for security F. Thus, given the private information $X = x$, the issuer chooses a security design F to solve the following:

$$\Gamma^P(x) = \max_{F(\cdot)} P[F] - \delta E[F(x+Z)]. \tag{8}$$

Denote by F_x the solution to Equation (8) corresponding to $X = x$. Given this solution, the price investors will pay should correspond to the expected payoff of the security conditional on the information revealed by the issuer's security choice. That is, the security design chosen by the issuer serves as a signal of the assets' value. In equilibrium,

$$P[F_X] = E[F_X(X+Z)|F_X]. \tag{9}$$

A signaling equilibrium corresponds to a simultaneous solution to Equations (8) and (9). In DeMarzo (2003b), I show that if attention is restricted to securities such that both the security payoff, $F(y)$, and the residual retained by the issuer, $y - F(y)$, are nondecreasing, there is a unique equilibrium satisfying the Intuitive Criterion of Cho and Kreps (1987). In this equilibrium, the optimal security design is a debt contract, with the face value of the debt depending on the private information X. That is, for each x, there is a face value $d(x)$ such that[23]

$$F_x(Y) = \min(d(x), Y).$$

Given this result, rewrite the issuer's problem [Equation (8)] as

$$\Gamma^P(x) = \max_d P(d) - \delta E[\min(d, x+Z)], \tag{10}$$

where $P(d)$ is the market price of debt with face value d.

We have the following immediate result:

Lemma 7. *For any demand function P, the issuer's profit Γ^P is continuous, decreasing, and convex in x. Also, the face value of the debt can be assumed to be decreasing in x.*

Proof. For any fixed d, the objective in Equation (10) is continuous, decreasing and convex in x, with a subgradient of $-\delta \Pr(d - x > Z)$. Since Γ^P is the upper-envelope of such functions, it is also continuous, decreasing and convex. Finally, optimal d can be chosen to be nonincreasing follows from the super-modularity of $E[\min(d, x+Z)]$. ■

[23] With multiple tranches, this is equivalent to tranches being prioritized, and the issuer selling the most senior tranches first up to a "hurdle" class; this is reasonably descriptive of actual practice.

Thus, the key property of convexity of the issuer's payoff holds for this setting as well. Also, since the issuer will optimally choose a face value of the debt that is decreasing in X, investors interpret large debt issues as a negative signal about the value of the assets. This leads to a separating equilibrium:

Lemma 8. *Given the asset pool Y, there is a unique separating equilibrium with $\Gamma^*(x) = (1-\delta)\ E[\min(d(x), x+Z)]$, $P^*(d(x)) = E[\min(d(x),\ x+Z)]$, and $d^*(x)$ determined by the differential equation:*

$$\frac{\partial}{\partial x} d^*(x) = -\frac{1}{(1-\delta)} \frac{\Pr(Z < d^*(x) - x)}{\Pr(Z > d^*(x) - x)}, \tag{11}$$

together with the boundary condition, $d^*(x_0) = \infty$.

The equilibrium described by Lemma 8 depends on two parameters: x_0, which affects the boundary condition, and the distribution of Z, written $\sim Z$, which affects the differential Equation (11). Thus, to compare equilibria across environments, define $\Gamma^*(x;\ x_0,\ \sim Z)$ and $d^*(x;\ x_0,\ \sim Z)$ to represent the solutions of Lemma 8 for the corresponding parameter values. The next result establishes properties of Γ^* analogous to Lemma 5:

Lemma 9. *The issuer's payoff is homogeneous of degree 1; that is, $a\Gamma^*(x;\ x_0,\ \sim Z) = \Gamma^*(ax;\ ax_0,\ \sim aZ)$. In addition, $\Gamma^*(x;\ x_0, \sim Z) = (1-\delta)x_0 + \Gamma^*(x - x_0;\ 0,\ \sim Z)$. Finally, $\Gamma^*(x;\ x_0,\ \sim Z) \le (1-\delta)\ x_0$, and the inequality is strict if $x > x_0$ and Z is nondegenerate.*

Having characterized the optimal security choice for a single asset, we now consider an issuer with n assets and compare the issuer's payoff from tranching a pool of assets versus selling and tranching each asset separately. The following result extends Theorem 2 through Theorem 4 to the case of ex post security design (unlimited tranching):

Theorem 5.

(*i*) *Suppose $\eta = 0$ so that the residual risk is idiosyncratic, and that Y_i has bounded second moments. Then as $n \to \infty$, the per-asset payoff from pooling and tranching approaches the theoretical maximum,*

$$\Gamma^*\left(\tfrac{1}{n}\sum_{i=1}^{n} X_i;\ \tfrac{1}{n}\sum_{i=1}^{n} x_{i0},\ \sim\tfrac{1}{n}\sum_{i=1}^{n} Z_i\right) \to (1-\delta)x_0,$$

so that pooling is optimal for sufficiently large n.

(*ii*) *Suppose $\varepsilon_i = 0$, so that the residual risk is common. Then pooling is suboptimal for any n.*

(iii) *In the setting of Theorem 4 (and the corollary), the issuer's payoff from pooling and tranching is increasing in the number of common factors in the information X_i.*

Thus, with either ex ante or ex post security design, pooling and tranching is optimal if the risk diversification effect dominates the information destruction effect.

5. Pooling by Uninformed Issuers

Sections 3 and 4 demonstrated that for an informed issuer, pure pooling is not optimal, though pooling and tranching may be optimal if there is sufficient diversification within the pool. In this section, I show that for an uninformed seller selling to both informed and uninformed buyers, pure pooling is optimal. This leads to the empirical prediction that only uninformed sellers should be observed selling pass-through pools.

Suppose that there exist firms, which I call "originators," that specialize in the marketing and other services associated with originating the assets. These firms do not have a comparative advantage in valuing these assets or holding them to maturity and instead plan to sell the assets at their market price and redeploy the capital for use in further origination projects.

Consider an originator holding an asset with future cash flow $Y = X + Z$. Assume that the originator does not specialize in valuing the assets, and thus does not know the information X.[24] As before, there are many potential risk-neutral uninformed buyers for the asset, who also do not know X, and who share the market discount rate of zero.

There are also potential informed investors who do know the realization of X. The informed investors have a higher cost of capital than uninformed, and so have valuation δX for the assets, for some $\delta < 1$.[25] I assume that buyers are anonymous, so that it is impossible for the seller to completely exclude these informed buyers from the market.

When the seller lists an asset for sale, uninformed buyers bid some price p for the asset. This price is such that the uninformed break-even on average. Because the uninformed compete with informed buyers who know X, they face an adverse selection problem. This leads to underpricing, $p < E[X]$, as in Rock's (1986) model of IPOs. The seller is also

[24] This does not require that the originator have no private information — indeed, the originator might have private information about the component Z of the cash flows from its knowledge of the original source of the assets. It could then signal this information through its issuance decision in the ways described in Sections 3 and 4. What is relevant for our purposes is that the informed buyers possess some information not known to the originator. In the interest of simplicity, I therefore assume the originator is uninformed.

[25] In general, δ depends on their ability to identify underpriced assets elsewhere, as I show in Section 6.

hurt by the adverse selection, since the valuation of informed buyers is strictly below that of uninformed buyers.

To determine the degree of underpricing, let $Q^u(X/p, p) \in [0, 1]$ be the total allocation to uninformed buyers, with Q^u weakly decreasing in the degree of underpricing, X/p, and, to capture cash constraints for the informed, weakly increasing in the size of the issue, p. This reduced form encompasses a variety of allocation mechanisms. For example, suppose the mechanism is a first price auction and informed buyers have discount factor δ and no cash constraint. Then the informed buy if $\delta X \geq p$, so that $Q^u(X/p, p) = 1[\delta X/p < 1]$, where $1[\cdot]$ is the indicator function. For a more complex example, if θ is the probability that a single informed trader is in the market, and if this trader has a cash constraint C, then

$$Q^u(X/p, p) = 1[\delta X/p < 1] + (1 - \theta \min(1, C/p))1[\delta X/p \geq 1].$$

Finally, suppose that a minimum level of under-pricing is necessary to attract informed buyers. To summarize:

Assumption 2. *Q^u is weakly decreasing in its first argument and is weakly increasing and continuous in its second argument. There exists $\beta > 1$ such that $Q^u(\beta, x_0) = 1$.*

Given Q^u, the equilibrium bid of the uninformed investors is the largest bid such that they earn nonnegative expected profits:

Lemma 10. *Suppose X is continuous with support $[x_0, x_1]$. Then the equilibrium uninformed bid is given by*

$$P^*[X] = \max\{p | E[Q^u(X/p, p)(X - p)] \geq 0\}.$$

Also, $P^*[X] = E[X]$ if and only if $Q^u(X/E[X], E[X]) = 1$ almost surely; otherwise, $x_0 < P^*[X] < E[X]$.

Now consider the incentives for pooling by an originator. In contrast to the results of Section 3 for an informed issuer, the following result shows that pure pass-through securities can be optimal for an uninformed originator, and can approach the first-best outcome.

Theorem 6. *Suppose X_i are independent, the assumptions of Lemma 10 hold, and $\frac{1}{n}\sum_i E[X_i] \to x > 0$ as $n \to \infty$. Then $\frac{1}{n}P^*[\sum_i X_i] \to x$.*

Thus, as the size of the pool grows large, the per-asset payoff to the originator approaches the assets' expected value (Figure 3). The intuition for this is straightforward. The adverse selection problem comes from the informed buyers' ability to purchase the best assets. Pooling reduces the precision of the selection the informed can make, as even the best pools will likely contain poor assets. This result provides an important motivation for pooling (in addition to transactions costs) in markets where

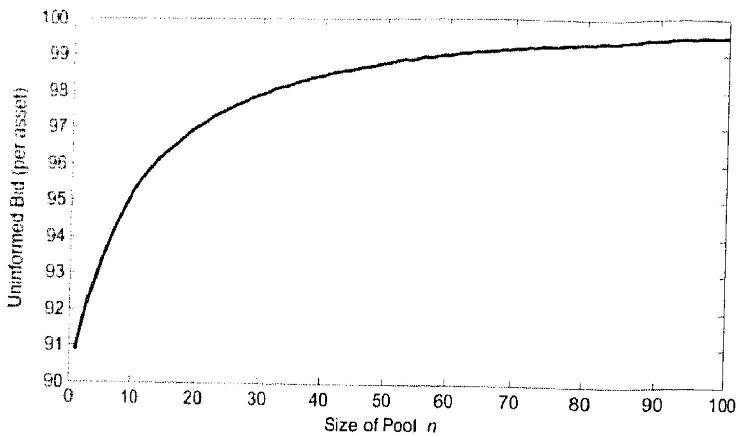

Figure 3
Uninformed bid (per asset) based on pool size
$E[Y_i] = 100$, $X_i = 90 + \xi_i$, ξ_i are i.i.d. exponential with mean 10. Informed have value δX, $\delta = 0.99$, and $Q^* = 1[\delta X < p]$.

traders other than the asset originators are likely to have the greatest expertise in valuing the assets.

6. A Dynamic Model of Informed Intermediation

The results of the previous sections demonstrate the following. While uninformed sellers can benefit by pooling assets prior to sale, informed sellers do not gain from pure pooling. On the other hand, informed sellers can benefit by pooling the assets and selling a debt tranche. Combining these results in the context of a dynamic model provides a theory of informed intermediation.

A simple motive for intermediation is implied by the returns to scale in the pooling and tranching process. An informed originator or holder of assets could sell the assets (or a tranche backed by them) directly to uninformed investors. Informed investment banks can add value, however, by acquiring these assets and forming even larger pools prior to tranching (Figure 4).

The second channel for intermediation begins with uninformed originators or asset holders. To minimize underpricing, uninformed originators will pool assets prior to sale. Informed investment banks will then purchase the best pools based on their superior information. After acquiring the assets, the investment bank pools them further and sells a senior tranche to investors in order to raise new capital for additional asset purchases.

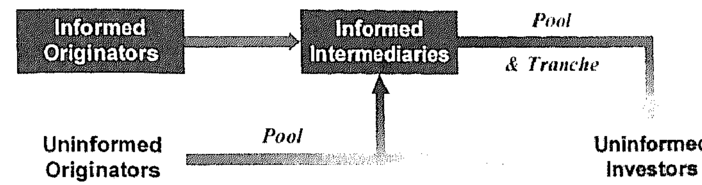

Figure 4
The flow of assets over time

In this section I build a simple dynamic model of this second channel for intermediation. While highly stylized, it demonstrates that the model is a consistent story of intermediation. Unlike standard models of informed trading which assume a buy and hold strategy for the informed, the model developed here illustrates the benefits of asset resale through securitization. In particular, I show how the ability to repackage assets allows the intermediary to leverage its capital and increase the returns from its information. The model also reveals how the key parameters of the static model, the worst-case information x_0 and the discount factor δ, arise endogenously from more primitive features of the market.

6.1 The timing

Consider a dynamic setting with the following timing. There is a single intermediary with access to a technology yielding private information. At the start of period t, the intermediary holds a portfolio of cash C_t and "old" securities with value O_t. At the start of period t, the origination market opens. The intermediary can use its available cash and superior information to purchase assets in the origination market that are underpriced.

After purchasing assets in the origination market, the intermediary holds both old securities (with value O_t) plus any new securities just acquired. Denote the full-information value of the new securities by N_t. In addition, the intermediary might have unused cash, denoted U_t, if the supply of new underpriced securities did not exceed its original cash balance C_t.

Once the origination market closes, the intermediary then has the opportunity to sell assets from its portfolio in the secondary market. By selling assets, the intermediary raises cash it can use in period $t+1$.

I assume that any private information the intermediary had regarding securities purchased prior to period t becomes public by the start of period t. Thus, in the sale phase, the intermediary will sell all the old securities O_t that were held at the start of the period for their full-information value. The new assets N_t will either be sold for cash or retained until the next period. Here there is a lemons problem since the intermediary holds private information about these securities. However, the intermediary

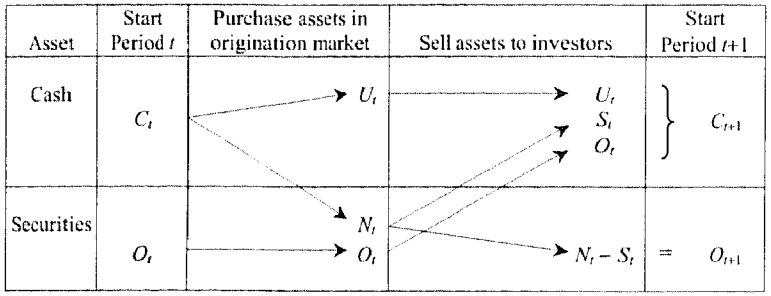

Figure 5
The timing of the model

also has a motive to trade, which is to raise additional cash to purchase assets next period. Thus, the signaling models of Sections 3 and 4 will determine the fraction of the value of the newly acquired assets that are sold and retained. Recall that in the separating equilibria of the previous analysis, the fraction sold, will be sold for its full information value, denoted S_t. Thus, the fraction retained has value $N_t - S_t$.[26] The intermediary then begins the next period with a portfolio of cash $C_{t+1} = U_t + S_t + O_t$ and old securities worth $O_{t+1} = N_t - S_t$ (Figure 5).

In the following sections, I employ the models of this article to determine the dynamics of this intermediation process.

6.2 Asset acquisition with cash constraints

Each period, uninformed originators sell assets in the origination market. Following Section 5, uninformed originators have an incentive to pool assets prior to sale, so each asset can be thought of as pool of even smaller assets. The pool sold by issuer i in period t has payoff $Y_{it} = X_{it} + Z_{it}$.

To provide a simple characterization of the equilibrium in the origination market, assume there is a continuum of originators $i \in [0, M_t]$ with associated measure μ. I assume that the intermediary's private information about each asset X_{it} is independent and identically distributed. In this case, it is natural to assume that

$$\mu\{i \in [0, M_t]: X_{it} \le x\} = M_t \Pr(X_t \le x),$$

[26] The intermediary might also have an incentive to issue securities backed by its *future* profit stream, rather than just its existing portfolio. Modeling this alternative is beyond the scope of this article. However, it is natural that such securities would be subject to even more extreme asymmetric information problems than the asset-backed securities considered. (For example, consider the distinction between secured and unsecured debt.) Serious moral hazard considerations would also be introduced. Thus, the intermediary would rely on asset-backed securities as a primary source of capital. This is consistent with actual practice: investment banks raise almost all of the cash used for acquiring new assets by selling or borrowing against existing assets in their portfolio.

where X_t has the same distribution as each of the X_{it}. Let X_t be continuously distributed. To simplify notation, I drop the time subscripts except where necessary.

Since the assets are ex ante identical, the uninformed bid a common price p for all assets. I assume the intermediary participates in a fraction $\theta < 1$ of the sales in the origination market.[27] In the absence of a budget constraint, the intermediary would purchase assets with $X_i > p$, at total cost $\theta M \Pr(X > p)\, p$. If this cost exceeds the intermediary's available cash C, the intermediary will buy assets above a critical quality x^c, where

$$x^c = \min x' \text{ such that } x' \ge p \quad \text{and} \quad \theta M \Pr(X > x')p \le C. \tag{12}$$

Anticipating this, uninformed investors realize that they receive assets disproportionately, that is, more of the lowest quality assets. Hence, the equilibrium bid p of the uninformed is the largest p satisfying the zero profit condition

$$E[(1 - \theta 1[X > x^c])(X - p)] = 0. \tag{13}$$

Together, Equations (12) and (13) determine the equilibrium values of x^c and p given θ and C/M, the amount of cash held by the intermediary relative to the size of the market. This extends the model of Section 5 to the continuum case.[28] Note the following comparative statistics for p and x^c as a function of the cash available to the intermediary.

Lemma 11. There exists $\bar{C} < \theta M E[X]$ such that for $C \le \bar{C}$, both x^c and p strictly decrease with C. For $C \ge \bar{C}$, $x^c = p = p_0$.

Figure 6 plots an example showing x^c and p as a function of C/M. Note that for $C/M \ge 72$, the cash constraint no longer binds and $x^c = p = 98.2$. However, for $C/M < 72$, the intermediary earns a positive return from additional cash balances since the marginal security is underpriced ($x^c > p$).

Figure 6 shows the worst-case value of the purchased portfolio, $\theta \Pr(X > x^c)\, x^c$, when valued at the minimum quality level x^c. I assume, as is true in this case, that this amount increases as the amount of cash spent increases and x^c decreases:

Assumption 3. *$\theta \Pr(X > x^c)\, x^c$ is decreasing in x^c for $x^c > p$.*

[27] Informed traders may be unaware of some fraction of the assets traded in the market, or originators may be able to prevent informed traders from participating with some probability (which is in originators' best interest as it reduces underpricing). The assumption $\theta < 1$ guarantees that the market price p is well behaved as the intermediary's expenditures approach the size of the market.

[28] If a single infinitesimal originator were to enter this market and place assets X' up for sale, the informed intermediary would purchase these assets if $X'/p' > x^c/p$. If we define $\delta = p/x^c$, this becomes $\delta X' > p'$, and $Q'' = 1 - \theta 1[\delta X' > p']$. The originator thus faces the same problem as in Section 5.

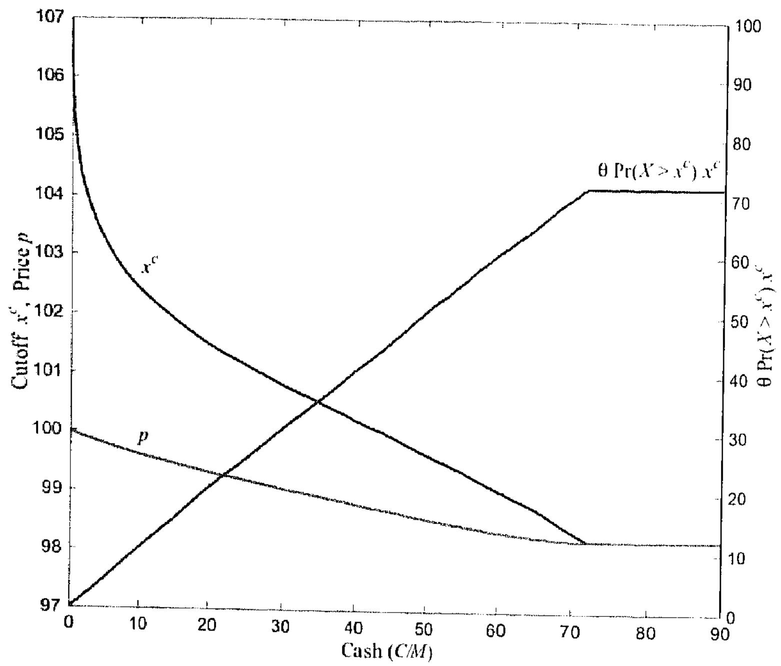

Figure 6
Equilibrium price p, informed cutoff x^c, and worst-case portfolio value versus informed size
$\theta = 0.90$, $E[X]=100$, and X is lognormal with volatility 2%.

Assumption 3 is a distributional assumption on the private information X that is satisfied by standard distributions if the volatility of X is not too large.[29] It implies that the minimal resale value of the intermediary's portfolio is increasing in its size.

6.3 Asset resale

At the end of the acquisition stage, the intermediary holds old assets (acquired in previous periods), new assets (just acquired), as well as any unused cash in the event $x^c = p$. The amount of unused cash is given by

$$U = C - \theta M \Pr(X > x^c)p. \tag{14}$$

The value of the new assets just purchased by the intermediary is given by

$$N = \theta ME[X1[X > x^c]] = C - U + M(E[X] - p). \tag{15}$$

[29] For example, it is satisfied if X is uniform on $[x_0, x_1]$ with $x_1 \leq 2x_0$, or for $X = x_0 + \xi$ with ξ exponential and $x_0 \geq E[\xi]$. In the case of a lognormal distribution, it holds as long as θ and σ are not too large. In the example here, we could raise θ to 0.999 or σ to 42% before the assumption is violated.

The first expression is the value of the assets purchased. The second expression follows from Equation (13): because the uninformed earn zero profits, the losses of the sellers $M(E[X] - p)$ correspond to the profits of the intermediary.

After acquiring new assets, the intermediary may resell them immediately or hold them until they are old and then resell them. The advantage of immediate resale is that the cash raised can be used to purchase new securities in the next period. The disadvantage is that the intermediary faces a lemons problem due to its private information.

First, consider the resale problem in the absence of asset securitization/tranching. The results in Section 3 establish that an informed intermediary should sell the assets individually. In that case there is a unique separating equilibrium, given in Lemma 2, in which an asset with value x is priced correctly and issued in quantity

$$Q^*(x; x_0, \delta) = \left(\frac{x}{x_0}\right)^{\frac{-1}{1-\delta}},$$

where x_0 is the worst-case information of the intermediary, and δ reflects the intermediary's preference for cash. Note that in this case, $x_0 = x^c$, the lowest quality asset purchased by the intermediary in the acquisition stage.

Given the parameter δ (we will see how δ is determined in the next section), the intermediary will raise cash from the resale of

$$S^1 = \theta ME[XQ^*(X; x^c, \delta)1[X > x^c]], \tag{16}$$

immediately by reselling the assets. The remaining fraction $1 - Q^*$ will be held for sale in the following period when the private information X is publicly known.

Alternatively, suppose tranching is possible. I assume the residual risk Z_i is diversifiable, so that it is eliminated in large pools. Thus, the results of Section 4 imply that it is optimal for the intermediary to form a pool of the purchased assets, and issue debt with face value $x_0 = x^c$ per asset. This debt is riskless, and so will sell for its face value of x^c, allowing the issuer to raise cash

$$S^* = \theta M \Pr(X > x^c)x^c. \tag{17}$$

The remaining junior tranche of the pool is sold in the following period for $N - S^*$. Comparing Equations (16) and (17), pooling and tranching benefit the intermediary by allowing it to raise more cash through immediate resale. That is, because $xQ^*(x; x^c, \delta) < x^c$ for $x > x^c$, we have $S^* > S^1$.

6.4 Growth through securitization

The analysis above leads to the following specification for the dynamic evolution of the intermediary. From Equations (12) and (13),

$$x_t^c = x^c(C_t, M_t), \quad p_t = p(C_t, M_t).$$

Combining this with Equations (14) and (15), it is possible to write

$$U_t = U(C_t, M_t), \quad N_t = N(C_t, M_t).$$

Finally, using either Equation (16) (no tranching, $S = S^1$) or Equation (17) (tranching, $S = S^*$), we have

$$S_t = S(\delta_t, C_t, M_t),$$

where $S = S^1$ or $S = S^*$ depending on the setting. This leads to the subsequent portfolio for the intermediary,

$$C_{t+1} = U_t + S_t + O_t, \quad O_{t+1} = N_t - S_t. \tag{18}$$

The only endogenous parameter not identified by the above system is δ_t, the intermediary's preference for cash. Suppose the intermediary raises additional cash in period t. The intermediary can use this cash to purchase assets with value x_{t+1}^c for price p_{t+1} and, because the incremental purchase is of the lowest quality asset, immediately resell the asset for price x_{t+1}^c.[30] Thus, a marginal dollar of cash generates a return of x_{t+1}^c/p_{t+1} so that the intermediary's preference for cash in period t is given by

$$\delta_t = p_{t+1}/x_{t+1}^c \leq 1. \tag{19}$$

Equation (19) can be combined with the above to yield the following fixed-point problem for δ_t,

$$\delta_t = \frac{p(C_{t+1}, M_{t+1})}{x^c(C_{t+1}, M_{t+1})} = \frac{p(U(C_t, M_t) + S(\delta_t, C_t, M_t) + O_t, M_{t+1})}{x^c(U(C_t, M_t) + S(\delta_t, C_t, M_t) + O_t, M_{t+1})}. \tag{20}$$

The solution to Equation (20) implies that[31]

$$\delta_t = \delta(C_t, O_t, M_t, M_{t+1}).$$

[30] This holds with or without tranching, since in a separating equilibrium the worst type bears no signaling cost.

[31] With tranching, $S = S^*$ does not depend on δ, hence the solution to Equation (20) is immediate. Without tranching, $S = S^1$ is decreasing in δ_t, and the fixed-point problem is nontrivial.

Combining all of the above, given some exogenous growth of the origination market, we may characterize the growth of the intermediary

$$C_t, O_t \xrightarrow[M_t, M_{t+1}]{} C_{t+1}, O_{t+1},$$

and derive the dynamics of market prices and pooling and tranching activity. The following result demonstrates the benefit of pooling and tranching for the intermediary:

Theorem 7. *Given initial assets, (C_0, O_0), the size of the intermediary, $C_T + O_T$, on date $T > 2$ will be larger when asset securitization (pooling and tranching) is possible than when assets are sold individually or there is no resale. The comparison is strict if $\delta_t < 1$ for some $t < T - 1$.*

Figure 7 shows the growth rate of the intermediary with pooling and tranching, individual resale, and no resale (buy and hold). Also shown is the marginal return on the worst assets purchased, x^c/p, showing the intermediary's preference for cash.

For example, Figure 7 implies that if the market growth rate is 3%, with pooling and tranching the intermediary will grow faster than the market until its size reaches a steady state of about 36% of the total market. In contrast, with individual resale, the intermediary grows to about 10% of

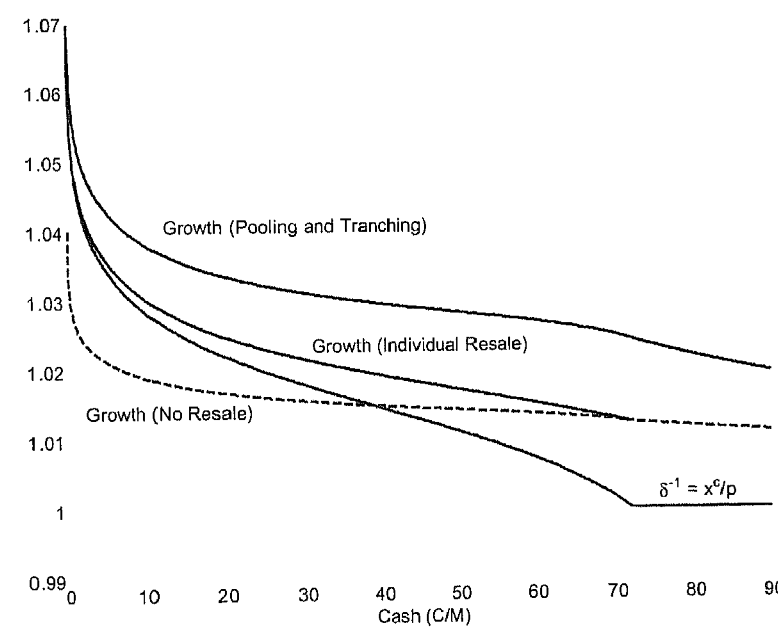

Figure 7
Intermediary growth rate versus size for different resale assumptions (parameters match Figure 6)

the market, and without resale, the steady state size of the intermediary is below 1% of the total market.[32]

7. Conclusion

The results of this article provide a theory of financial intermediation based upon intermediaries having private information regarding asset values. Due to its information advantage, the intermediary has the ability to identify high quality and therefore underpriced assets in the origination market. The intermediary can therefore profit by buying and holding these assets. Of course, this creates an adverse selection problem in the origination market, implying that in equilibrium these assets will be priced at a discount. In order to mitigate this problem, originators have an incentive to pool the assets prior to selling them. Pooling reduces the intermediary's ability to purchase the assets that are most underpriced.

Once the intermediary has purchased assets in the origination market, it can hold the assets to maturity. The intermediary, however, would prefer to liquidate the assets at their true value to raise cash to use for future asset purchase opportunities. That is, the intermediary wishes to leverage its available capital to exploit its information for as many deals as possible. Unfortunately, because the intermediary has private information regarding the assets, it faces a lemons problem if it attempts to sell the assets for cash. This lemons problem leads to a natural signaling equilibrium in which the intermediary signals the value of the assets by its willingness to retain some portion of the cash flows. I show that if only pure pass-through securities can be sold, the intermediary finds it optimal to sell the assets individually rather than as a pool. However, if nonlinear asset-backed securities can be issued, and if the intermediary holds enough assets, it can be optimal to pool the assets and issue a debt-like security that is backed by the pool. Asset securitization allows the intermediary to leverage its capital more efficiently and increase the returns associated with its private information.

The incentive for pooling and tranching by an issuer is shown to depend on several factors. First, pooling has an information destruction effect that is costly for the intermediary. This effect is reduced if the intermediary's private information is positively correlated across the assets. Second, the gains from issuing a debt tranche are enhanced if the pool has lower residual risk. This risk diversification benefit is therefore reduced if the residual risks of the assets are positively correlated. Thus, pooling and tranching is most effective for assets for which the private information is general and the residual risks are specific.

[32] Note that without resale, even though the marginal return is above 5% when cash is below 1%, this return is realized over 2 periods (assets are not sold until information is public). Thus the annualized growth rate is below the marginal return. With either type of resale, since the intermediary can always sell for at least the worst-case value x, the growth rate cannot fall below the marginal return δ^{-1}. Note also that the growth rate with no resale and individual resale coincide for $\delta = 1$ since then the intermediary sells only the worst assets (which have measure zero). With pooling and tranching, however, the intermediary can extract x from each asset (even those with higher value).

Appendix

Proof of Lemma 3. Given X_{-n}, consider the sale of asset n. By the initial assumptions, the conditional support of X_n is an interval with greatest lower bound x_{n0}. Thus, from the previous results, there is a unique separating equilibrium for the sale of asset n, with the issuer's profit given by $\pi(X_n/x_{n0})x_{n0}$. Now consider the sale of asset $n-1$. Since the issuer's profit in the sale of asset n does not depend on the outcome of the sale of asset $n-1$, the issuer's problem is unchanged. The proof thus proceeds by induction. ∎

Proof of Lemma 5. For any security design F for Y, define the security $\hat{F}$ for aY by $\hat{F}(y) = aF(y/a)$. Then $\hat{F}(aY) = aF(Y)$. Hence $\hat{f} = af$, and homogeneity follows immediately from Equation (4).

Next note that since π is decreasing, $\pi(f/f_0)f_0 \le \pi(1)f_0 = (1-\delta)f_0$. The inequality then follows since $f = E[F(Y)\,|\,X] \le E[Y\,|\,X] = X$, so that $f_0 \le x_0$.

For strictness, suppose $f_0 = x_0$. Then $F(x_0+Z) = x_0+Z$ almost surely. Thus, F is strictly increasing on the support of x_0+Z. Since F is nondecreasing everywhere, if $x > x_0$ then $F(x+Z) > F(x_0+Z)$ for Z in the interior of its support, or almost surely. Hence $f/f_0 > 1$ almost surely. Since π is strictly decreasing, the result follows. ∎

Proof of Theorem 2. Define $H_n(d,x) = E[\min(d, x + \frac{1}{n}\sum_{i=1}^{n}\varepsilon_i)]$. H_n is continuous and increasing, and by the Law of Large Numbers, $H_n(d,x) \to \min(d,x)$ as $n \to \infty$.

First consider $d = x_0$. Then $f_0^d = H_n(d, x_0^n/n) \to \min(d,x_0) = x_0$. Since $d \ge f^d = H_n(d, X^n/n) \ge H_n(d, x_0^n/n)$, we also have $f^d \to x_0$ almost surely. Hence, $E[\pi(f^d/f_0^d)f_0^d] \to \pi(1)x_0 = (1-\delta)x_0$. Since this is an upper bound for G by Lemma 5, we conclude that $G[\frac{1}{n}\sum_{i=1}^{n} Y_i] \to (1-\delta)x_0 > \frac{1}{n}\sum_{i=1}^{n} G[Y_i]$.

To show that D^* must converge to x_0, consider $d < x_0$. In that case, $f_0^d \to d$, so that $\overline{\lim}_{n\to\infty} E[\pi(f^d/f_0^d)f_0^d] \le (1-\delta)d$, which is suboptimal. Next consider $d > x_0$. In that case, $f_0^d \to x_0$, so it remains to show that $\overline{\lim}_{n\to\infty} E[\pi(f^d/f_0^d)] < (1-\delta)$.

Since $G[Y_i] \ge E[\pi(X_i/x_{i0})x_{i0}] \ge \pi(E[X_i]/x_{i0})x_{i0}$, Equation (7) implies that $\underline{\lim}_{n\to\infty} \frac{1}{n} \Sigma_i E[X_i] > x_0$. Thus, there exists N and $\gamma \in (0, d-x_0)$ such that for all $n > N$, $E[X^n]/n > x_0 + \gamma$. By hypothesis, X^n/n has bounded second moment, so that[33] $\Pr(X^n/n > x_0 + \gamma/2) > \lambda$ for some $\lambda > 0$. Also, there exists $N' > N$ such that for all $n > N'$, $H_n(d, x_0 + \gamma/2) \ge x_0 + \gamma/4$ and $H_n(d, x_0^n) \le x_0 + \gamma/6$. Thus, for all $n > N'$, $\Pr\left(\pi(f^d/f_0^d) \le \pi\left(\frac{x_0+\gamma/4}{x_0+\gamma/6}\right)\right) > \lambda$, which proves the result. ∎

Proof of Theorem 3. Fix an n and let $d = D^*[Y^n/n]$, the optimal debt level if the assets are pooled. Next define $h(x) = E[\min(d, x+\eta)]$, and note that h is concave. Then for any realization x_i, Jensen's inequality implies $h(\sum_i x_i/n) \ge \sum_i h(x_i)/n$. Since π is decreasing, this implies

$$\pi\left(\frac{h(\frac{1}{n}\sum_i x_i)}{h(x_0)}\right) \le \pi\left(\frac{1}{n}\sum_i \frac{h(x_i)}{h(x_0)}\right) \le \frac{1}{n}\sum_i \pi\left(\frac{h(x_i)}{h(x_0)}\right),$$

[33] This follows from the fact that if $E[A] \ge a$ and $E[A^2] \le b$, then for $c \in [0, a]$, $\Pr(A \ge c) \ge (a-c)^2/b$.

where the last inequality follows from the convexity of π. Thus,

$$G\left[\frac{1}{n}\sum_i Y_i\right] = E\left[\pi\left(\frac{h(\frac{1}{n}\sum_i x_i)}{h(x_0)}\right)h(x_0)\right] \le \frac{1}{n}\sum_i E\left[\pi\left(\frac{h(x_i)}{h(x_{i0})}\right)h(x_{i0})\right] \le \frac{1}{n}\sum_i G[Y_i]$$

where the last inequality follows since $d \neq D^*[Y_i]$ in general. ■

Proof of Theorem 4. Define $h(d, x) = E[\min(d, x+\eta+\sum\varepsilon_i/n)]$, and note that h is concave and increasing. Next define $Q(d, x, x_0) = \pi(h(d, x)/h(d, x_0))h(d, x_0)$. Since π is decreasing and convex, Q is also decreasing and convex in x. Therefore,

$$E\left[Q\left(d, \frac{1}{n}\sum_i X_i, \frac{1}{n}\sum_i x_{i0}\right)\right] \ge E\left[Q\left(d, \frac{1}{n}\sum_i \hat{X}_i, \frac{1}{n}\sum_i x_{i0}\right)\right],$$

which proves the result. ■

Proof of Lemma 8. In any sequential equilibrium, the "worst type" x_0 behaves according to the first-best, which in the context of this model is to sell all of the assets to the investors. This is equivalent to issuing 100% of the equity interest in the assets, or equivalently debt with a face value equal to or greater than the maximum possible payoff. It is then straightforward to check that the model satisfies the standard single crossing condition and that the above differential equation does indeed determine an equilibrium. Uniqueness follows by similar arguments to Mailath (1987). ■

Proof of Lemma 9. Define $d(x) = ad^*(x/a; x_0, \sim Z)$. Then $d(ax_0) = \infty$ and

$$\begin{aligned} d'(x) &= a\frac{1}{a}d^{*\prime}(x/a; x_0, \sim Z) = -\frac{1}{(1-\delta)}\frac{\Pr(Z < d^*(x/a; x_0, \sim Z) - x/a)}{\Pr(Z > d^*(x/a; x_0, \sim Z) - x/a)} \\ &= -\frac{1}{(1-\delta)}\frac{\Pr(aZ < d(x) - x)}{\Pr(aZ > d(x) - x)}. \end{aligned}$$

Hence, $d(x) = d^*(x; ax_0, \sim aZ)$. Then we have,

$$\begin{aligned} \Gamma^*(ax;\ ax_0, \sim aZ) &= (1-\delta)E[\min(d(ax), ax + aZ)] \\ &= (1-\delta)E[\min(ad^*(x;\ x_0, \sim Z), ax + aZ)] \\ &= a\Gamma^*(x;\ x_0, \sim Z). \end{aligned}$$

Next define $d_0(x) = d^*(x + x_0; x_0, \sim Z) - x_0$. Then $d_0(0) = \infty$ and

$$\begin{aligned} d_0'(x) = d^{*\prime}(x + x_0;\ x_0, \sim Z) &= -\frac{1}{(1-\delta)}\frac{\Pr(Z < d^*(x + x_0; x_0, \sim Z) - (x + x_0))}{\Pr(Z > d^*(x + x_0; x_0, \sim Z) - (x + x_0))} \\ &= -\frac{1}{(1-\delta)}\frac{\Pr(Z < d_0(x) - x)}{\Pr(Z > d_0(x) - x)}. \end{aligned}$$

Thus, $d_0(x) = d^*(x; 0, \sim Z)$. Therefore,

$$\begin{aligned} \Gamma^*(x - x_0;\ 0, \sim Z) &= (1-\delta)E[\min(d_0(x - x_0), x - x_0 + Z)] \\ &= (1-\delta)E[\min(d^*(x; x_0, \sim Z) - x_0, x - x_0 + Z)] \\ &= \Gamma^*(x;\ x_0, \sim Z) - (1-\delta)x_0. \end{aligned}$$

For the bound on Γ^*, note that

$$\Gamma^*(x_0;\ x_0, \sim Z) = (1-\delta)E[\min(d^*(x_0; x_0, \sim Z), x_0 + Z)] = (1-\delta)E[x_0 + Z] = (1-\delta)x_0,$$

and by Lemma 7, Γ^* is decreasing in x. For strictness, note that if Γ^* at x_0 does not strictly decrease, by Lemma 7 it must be constant. Therefore, $E[\min(d^*(x; x_0, \sim Z), x + Z)] = x_0$. For $x > x_0$, this implies that $\Pr(x_0 + Z > d^*(x; x_0, \sim Z)) > 0$. But this implies type x_0 would prefer to issue $d^*(x; x_0, \sim Z)$, violating incentive compatibility. ■

Proof of Theorem 5. For case (i), suppose the issuer sells debt with a (per-asset) face value of $d = \frac{1}{n}x_0^n$. In any sequential equilibrium, this debt will sell for a (per-asset) price of at least $p_n = E[\min(\frac{1}{n}x_0^n, \frac{1}{n}x_0^n + \frac{1}{n}\varepsilon^n)]$. Therefore,

$$\begin{aligned} \Gamma^*\left(\tfrac{1}{n}X^n;\ \tfrac{1}{n}x_0^n, \sim\tfrac{1}{n}Z^n\right) &\ge p_n - \delta E\left[\min\left(\tfrac{1}{n}x_0^n, \tfrac{1}{n}X^n + \tfrac{1}{n}\varepsilon^n\right)\right] \\ &\ge p_n - \delta\tfrac{1}{n}x_0^n \\ &= (1-\delta)\tfrac{1}{n}x_0^n + E\left[\min(0, \tfrac{1}{n}\varepsilon^n)\right] \\ &\ge (1-\delta)\tfrac{1}{n}x_0^n - \tfrac{1}{2\sqrt{n}}\bar{\sigma} \end{aligned}$$

where the last inequality follows from an application of Cauchy–Schwarz and $\text{var}(\varepsilon_i) \le \bar{\sigma}^2$. For an upper bound, recall from Lemma 9 that the per-asset payoff to the issuer is bounded above by $(1-\delta)x_0^n/n$, which has the same limit.

For case (ii), if the issuer pools the assets the resulting payoff is $\Gamma^*(\sum_i X_i/n;\ \sum_i x_{i0}/n, \sim\eta)$. From Lemma 9, this is equal to $\Gamma^*(\sum_i(X_i - x_{i0})/n;\ 0, \sim\eta) + (1-\delta)\sum_i x_{i0}/n$. Next, by Lemma 7, Γ^* is convex, so that

$$\begin{aligned} \Gamma^*\left(\frac{1}{n}\sum_{i=1}^n(X_i - x_{i0}); 0, \sim\eta\right) + (1-\delta)\left(\frac{1}{n}\sum_{i=1}^n x_{i0}\right) &\le \frac{1}{n}\sum_{i=1}^n \Gamma^*(X_i - x_{i0}; 0, \sim\eta) + (1-\delta)x_{i0} \\ &= \frac{1}{n}\sum_{i=1}^n \Gamma^*(X_i; x_{i0}, \sim\eta). \end{aligned}$$

Hence, the payoff from the sales of separate tranches exceeds the payoff from tranching a single asset pool. Finally, case (iii) follows immediately from the convexity of Γ^*. ■

Proof of Lemma 10. The expected profits of an uninformed bidder with bid p is given by $E[Q^u(X-p)]$. Since $|Q^u(X-p)| \le |X-p|$, dominated convergence and the continuity assumptions on Q^u imply that $E[Q^u(X-p)]$ is continuous in p. Since profits are positive (negative) for $p = x_0$ ($p = x_1$), P^* is interior and earns zero profits for the uninformed bidders, and is an equilibrium since any higher bid earns negative profits. Finally, $E[Q^u(X-p)] \ge 0$ implies

$$p \le E[X] + \frac{\text{Cov}(Q^u, X)}{E[Q^u]} \le E[X]$$

where the second inequality follows since Q^u is weakly decreasing in X, and the inequality is strict if Q^u is not constant. ■

Proof of Theorem 6. First note that $\frac{1}{n}P^*\left[\sum_i X_i\right] = P^*\left[\frac{1}{n}\sum_i X_i\right]$. For any $\gamma \in (1, \beta)$, let $p_n = \beta^{-1}\gamma E\left[\frac{1}{n}X^n\right]$. From the Schwarz inequality,

$$E[A|B] \ge E[A] - \sigma_A\sqrt{\frac{1}{\Pr(B)} - 1}.$$

Since $\text{Var}[X^n/n] = \frac{1}{n^2}\sum_{i=1}^n \text{Var}[X_i] \le \frac{1}{n}\bar{\sigma}^2$ this implies,

$$E\left[\tfrac{1}{n}X^n \middle| \tfrac{1}{n}X^n \le \beta p_n\right] \ge E\left[\tfrac{1}{n}X^n\right] - \frac{1}{\sqrt{n}}\bar{\sigma}\sqrt{\frac{1}{\Pr(\frac{1}{n}X^n \le \beta p_n)} - 1}. \qquad (21)$$

Since $\gamma > 1$, by the Weak Law of Large Numbers,

$$\Pr\left(\tfrac{1}{n}X^n \le \beta p_n\right) = \Pr\left(\tfrac{1}{n}(X^n - E[X^n]) \le (\gamma - 1)\tfrac{1}{n}E[X^n]\right) \to 1.$$

Thus, Equation (21) implies $E[\frac{1}{n}X^n | \frac{1}{n}X^n \leq \beta p_n] \to x$. Hence, for sufficiently large n, since $p_n \to \beta^{-1}\gamma x < x$,

$$p_n \leq E\left[\frac{1}{n}X^n \middle| \frac{1}{n}X^n \leq \beta p_n\right] \leq \frac{E[Q^n(\frac{1}{n}X^n, p_n)X]}{E[Q^n(\frac{1}{n}X^n, p_n)]},$$

where the last inequality follows since $Q^n(X/p, p) = 1[X \leq \beta p] + 1[X > \beta p]Q^n(X/p, p)$. Thus, at price p_n, uninformed investors earn a nonnegative profit. This implies that $E[\frac{1}{n}X^n] \geq P^*[\frac{1}{n}X^n] \geq p_n$, and therefore $\lim_{n \to \infty} P^*[\frac{1}{n}X^n] \in (\beta^{-1}\gamma x, x)$. Since this is true for all $\gamma \in (1, \beta)$, we have $P^*[\frac{1}{n}X^n] \to x$. ■

Proof of Lemma 11. Differentiating Equation (13) implies p decreases (increases) as x^c decreases for $x^c > (<) p$. Continuity of p in x^c follows from the continuity of X. Thus there exists p_0 such that $x^c \geq p$ for $x^c \geq p_0$. Since the cash constraint in Equation (12) is relaxed with an increase in C, both x^c and p strictly decrease with C until $x^c = p = p_0$. This occurs for $C = \bar{C} = \theta M \Pr(X > p_0) p_0 < \theta M p_0 < \theta M E[X]$. ■

Proof of Theorem 7. Given the same initial cash, N_0 is equal in both cases. However, since $S^* > S^1$, the intermediary has more cash at date 1 with pooling and tranching. If $\delta_0 < 1$, $x^c_1 > p_1$, and so p_1 is decreasing in the amount of cash held by the intermediary from Lemma 11. Thus, N_1 will be higher with pooling and tranching, and thus so will $C_2 + O_2$. Also, from Assumption 3, the decrease in x^c_1 implies that C_2 will also be higher with pooling and tranching. Thus, on date 2 the intermediary has both higher cash and higher total assets with pooling and tranching. This argument can be repeated at each future date. ■

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