Premium Principles

Introduction

Loosely speaking, a *premium principle* is a rule for assigning a premium to an insurance risk. In this article, we focus on the premium that accounts for the monetary payout by the insurer in connection with insurable losses plus the risk loading that the insurer imposes to reflect the fact that experienced losses rarely, if ever, equal expected losses. In other words, we ignore premium loadings for expenses and profit.

In this article, we describe three methods that actuaries use to develop premium principles. The distinction among these methods is somewhat arbitrary, and a given premium principle might arise from more than one of the methods.

We call the first method the *ad hoc method* because within it, an actuary defines a (potentially) reasonable premium principle, then she determines which (if any) of a list of desirable properties her premium principle satisfies. For example, the Expected Value Premium Principle, in which the premium equals the expected value times a number greater than or equal to one, is rather ad hoc, but it satisfies some nice properties. Indeed, it preserves affine transformations of the random variable and is additive. However, it does not satisfy all desirable properties, as we will see in the section 'Catalog of Premium Principles: The Ad Hoc Method'.

A more rigorous method is what we call the *characterization method* because within it, an actuary specifies a list of properties that she wants the premium principle to satisfy and then finds the premium principle (or set of premium principles) determined by those properties. Sometimes, an actuary might not necessarily characterize the set of premium principles that satisfy the given list of properties but will find one premium principle that does. Finding only one such premium principle is a weaker method than the full-blown characterization method, but in practice, it is often sufficient. An example of the weaker method is to look for a premium principle that is scale equivariant, and we will see in the section 'Catalog of Premium Principles: The Ad Hoc Method' that the

standard deviation premium principle is scale equivariant but it is not the only such premium principle.

Perhaps the most rigorous method that an actuary can use to develop a premium principle is what we call the *economic method*. Within this method, an actuary adopts a particular economic theory and then determines the resulting premium principle. In the section 'The Economic Method', we will see that the important Esscher Premium Principle is such a premium principle [1, 2].

These methods are not mutually exclusive. For example, the Proportional Hazards Premium Principle [3] first arose when Wang searched for a premium principle that satisfied layer additivity; that is, Wang used a weak form of the characterization method. Then, Wang, Young, and Panjer [4] showed that the Proportional Hazards Premium Principle can be derived through specifying a list of properties and showing that the Proportional Hazards Premium Principle is the *only* premium principle that satisfies the list of properties. Finally, the Proportional Hazards Premium Principle can also be grounded in economics via Yaari's dual theory of risk (*see* **Risk Utility Ranking**) [5].

Some premium principles arise from more than one method. For example, an actuary might have a particular property (or a list of properties) that she wants her premium principle to satisfy, she finds such a premium principle, then later discovers that her premium principle can be grounded in an economic premium principle. In the sections 'The Characterization Method' and 'The Economic Method', we will see that Wang's premium principle can arise from both the characterization and economic methods.

In the section 'Properties of Premium Principles', we catalog desirable properties of premium principles for future reference in this article. In the section 'Catalog of Premium Principles: The Ad Hoc Method', we define some premium principles and indicate which properties they satisfy. Therefore, this section is written in the spirit of the ad hoc method. In the section 'The Characterization Method', we demonstrate the characterization method by specifying a list of properties and by then determining which premium principle exactly satisfies those properties. As for the economic method, in the section 'The Economic Method', we describe some economic theories adopted by actuaries and determine the resulting premium principles. In the 'Summary', we conclude this review article.

Properties of Premium Principles

In this section, we list and discuss desirable properties of premium principles. First, we present some notation that we use throughout the paper. Let χ denote the set of nonnegative random variables on the probability space (Ω, F, P); this is our collection of insurance-loss random variables – also called insurance risks. Let X, Y, Z, etc. denote typical members of χ. Finally, let H denote the premium principle, or function, from χ to the set of (extended) nonnegative real numbers. Thus, it is possible that $H[X]$ takes the value ∞. It is possible to extend the domain of a premium principle H to include possibly negative random variables. That might be necessary if we were considering a general loss random variable of an insurer, namely, the payout minus the premium [6]. However, in this review article, we consider only the insurance payout and refer to that as the insurance loss random variable.

1. *Independence*: $H[X]$ depends only on the (de)cumulative distribution function of X, namely S_X, in which $S_X(t) = P\{\omega \in \Omega : X(\omega) > t\}$. That is, the premium of X depends only on the tail probabilities of X.

   This property states that the premium depends only on the monetary loss of the insurable event and the probability that a given monetary loss occurs, not the cause of the monetary loss.

2. *Risk loading*: $H[X] \geq EX$ for all $X \in \chi$.

   Loading for risk is desirable because one generally requires a premium rule to charge at least the expected payout of the risk X, namely EX, in exchange for insuring the risk. Otherwise, the insurer will lose money on average.

3. *No unjustified risk loading*: If a risk $X \in \chi$ is identically equal to a constant $c \geq 0$ (almost everywhere), then $H[X] = c$.

   In contrast to Property 2 (Risk loading), if we know for certain (with probability 1) that the insurance payout is c, then we have no reason to charge a risk loading because there is no uncertainty as to the payout.

4. *Maximal loss (or no rip-off)*: $H[X] \leq \text{ess sup}[X]$ for all $X \in \chi$.

5. *Translation equivariance (or translation invariance)*: $H[X + a] = H[X] + a$ for all $X \in \chi$ and all $a \geq 0$.

If we increase a risk X by a fixed amount a, then Property 5 states that the premium for $X + a$ should be the premium for X increased by that fixed amount a.

6. *Scale equivariance (or scale invariance)*: $H[bX] = bH[X]$ for all $X \in \chi$ and all $b \geq 0$.

   Note that Properties 5 and 6 imply Property 3 as long as there exists a risk Y such that $H[Y] < \infty$. Indeed, if $X \equiv c$, then $H[X] = H[c] = H[0 + c] = H[0] + c = 0 + c = c$. We have $H[0] = 0$ because $H[0] = H[0Y] = 0H[Y] = 0$. Scale equivariance is also known as homogeneity of degree one in the economics literature. This property essentially states that the premium for doubling a risk is twice the premium of the single risk. One usually uses a no-**arbitrage** argument to justify this rule. Indeed, if the premium for $2X$ were *greater* than twice the premium of X, then one could buy insurance for $2X$ by buying insurance for X with two different insurers, or with the same insurer under two policies. Similarly, if the premium for $2X$ were *less* than twice the premium of X, then one could buy insurance for $2X$, sell insurance on X and X separately, and thereby make an arbitrage profit. Scale equivariance might not be reasonable if the risk X is large and the insurer (or insurance market) experiences surplus constraints. In that case, we might expect the premium for $2X$ to be greater than twice the premium of X, as we note below for Property 9. Reich [7] discussed this property for several premium principles.

7. *Additivity*: $H[X + Y] = H[X] + H[Y]$ for all X, $Y \in \chi$.

   Property 7 (Additivity) is a stronger form of Property 6 (Scale equivariance). One can use a similar no-arbitrage argument to justify the additivity property [8–10].

8. *Subadditivity*: $H[X + Y] \leq H[X] + H[Y]$ for all $X, Y \in \chi$.

   One can argue that subadditivity is a reasonable property because the no-arbitrage argument works well to ensure that the premium for the sum of two risks is not greater than the sum of the individual premiums; otherwise, the buyer of insurance would simply insure the two risks separately. However, the no-arbitrage argument that asserts that $H[X + Y]$ cannot be less than $H[X] + H[Y]$ fails because it is generally not possible for the buyer of insurance to sell insurance for the two risks separately.

9. *Superadditivity*: $H[X + Y] \geq H[X] + H[Y]$ for all $X, Y \in \chi$.

Superadditivity might be a reasonable property of a premium principle if there are surplus constraints that require that an insurer charge a greater risk load for insuring larger risks. For example, we might observe in the market that $H[2X] > 2H[X]$ because of such surplus constraints.

Note that both Properties 8 and 9 can be weakened by requiring only $H[bX] \leq bH[X]$ or $H[bX] \geq bH[X]$ for $b > 0$, respectively. Next, we weaken the additivity property by requiring additivity only for certain insurance risks.

10. *Additivity for independent risks*: $H[X + Y] = H[X] + H[Y]$ for all $X, Y \in \chi$ such that X and Y are independent.

Some actuaries might feel that Property 7 (Additivity) is too strong and that the no-arbitrage argument only applies to risks that are independent. They, thereby, avoid the problem of surplus constraints for dependent risks.

11. *Additivity for comonotonic risks*: $H[X + Y] = H[X] + H[Y]$ for all $X, Y \in \chi$ such that X and Y are comonotonic (*see* **Comonotonicity**).

Additivity for comonotonic risks is desirable because if one adopts subadditivity as a general rule, then it is unreasonable to have $H[X + Y] < H[X] + H[Y]$ because neither risk is a *hedge* (*see* **Hedging and Risk Management**) against the other, that is, they move together [5]. If a premium principle is additive for comonotonic risks, then is it layer additive [11]. Note that Property 11 implies Property 6, (Scale equivariance), if H additionally satisfies a continuity condition.

Next, we consider properties of premium rules that require that they preserve common **orderings of risks**.

12. *Monotonicity*: If $X(\omega) \leq Y(\omega)$ for all $\omega \in \Omega$, then $H[X] \leq H[Y]$.

13. *Preserves first stochastic dominance (FSD) ordering*: If $S_X(t) \leq S_Y(t)$ for all $t \geq 0$, then $H[X] \leq H[Y]$.

14. *Preserves stop-loss ordering (SL) ordering*: If $E[X - d]_+ \leq E[Y - d]_+$ for all $d \geq 0$, then $H[X] \leq H[Y]$.

Property 1, (Independence), together with Property 12, (Monotonicity), imply Property 13, (Preserves FSD ordering) [4]. Also, if H preserves SL ordering, then H preserves FSD ordering because stop-loss ordering is weaker [12]. These orderings are commonly used in actuarial science to order risks (partially) because they represent the common orderings of groups of decision makers; see [13, 14], for example.

Finally, we present a technical property that is useful in characterizing certain premium principles.

15. *Continuity*: Let $X \in \chi$; then, $\lim_{a \to 0^+} H[\max(X - a, 0)] = H[X]$, and $\lim_{a \to \infty} H[\min(X, a)] = H[X]$.

Catalog of Premium Principles: The Ad Hoc Method

In this section, we list many well-known premium principles and tabulate which of the properties from the section 'Properties of Premium Principles' they satisfy.

A. *Net Premium Principle*: $H[X] = EX$.

This premium principle does not load for risk. It is the first premium principle that many actuaries learn; see, for example [6]. It is widely applied in the literature because actuaries often assume that risk is essentially nonexistent if the insurer sells enough identically distributed and independent policies [15–23].

B. *Expected Value Premium Principle*: $H[X] = (1 + \theta) EX$, for some $\theta > 0$.

This premium principle builds on Principle A, the Net Premium Principle, by including a proportional risk load. It is commonly used in insurance economics and in **risk theory**; see, for example [6]. The expected value principle is easy to understand and to explain to policyholders.

C. *Variance Premium Principle*: $H[X] = EX + \alpha \text{Var} X$, for some $\alpha > 0$.

This premium principle also builds on the Net Premium Principle by including a risk load that is proportional to the variance of the risk. Bühlmann [24, Chapter 4] studied this premium principle in detail. It approximates the premium that one obtains from the principle of equivalent utility (or zero-utility) (*see* **Utility Theory**), as we note below. Also, Berliner [25] proposed a risk measure that is

an alternative to the variance and that can be used in this premium principle.

D. *Standard Deviation Premium Principle*: $H[X] = EX + \beta\sqrt{\mathrm{Var}\,X}$, for some $\beta > 0$.

This premium principle also builds on the Net Premium Principle by including a risk load that is proportional to the standard deviation of the risk. Bühlmann [24, Chapter 4] also considered this premium principle and mentioned that it is used frequently in property (*see* **Property Insurance – Commercial Only**; **Property Insurance – Personal**) and **casualty insurance** (*see* **Nonlife Insurance**). As we note below, Wang's Premium Principle reduces to the Standard Deviation Premium Principle in many cases. Also, Denneberg [26] argued that one should replace the standard deviation with absolute deviation in calculating premium. Finally, Schweizer [27] and Møller [28] discussed how to adapt the Variance Premium Principle and the Standard Deviation Principle to pricing risks in a dynamic financial market.

E. *Exponential Premium Principle*: $H[X] = (1/\alpha) \times \ln E[e^{\alpha X}]$, for some $\alpha > 0$.

This premium principle arises from the principle of equivalent utility (Principle H below) when the utility function is exponential [29, 30]. It satisfies many nice properties, including additivity with respect to independent risks. Musiela and Zariphopoulou [31] adapted the Exponential Premium Principle to the problem of pricing financial securities in an **incomplete market**. Young and Zariphopoulou [32], Young [33], and Moore and Young [34] used this premium principle to price various insurance products in a dynamic market.

F. *Esscher Premium Principle*: $H[X] = (E[Xe^Z]/E[e^Z])$, for some random variable Z.

Bühlmann [1, 2] derived this premium principle when he studied risk exchanges (*see* **Equilibrium Pricing Model**); also see [35, 36]. In that case, Z is a positive multiple of the aggregate risk of the exchange market. Some authors define the Esscher Premium Principle with $Z = hX$, $h > 0$. For further background on the Esscher Premium Principle, which is based on the **Esscher transform**, please read [37, 38]. Also, see the research of Gerber and Shiu for more information about how to apply the Esscher Premium Principle in mathematical finance [39, 40]. The Esscher Premium Principle (as given here in full generality) is also referred to as the Exponential

Tilting Premium Principle. Heilmann [41] discussed the case for which $Z = f(X)$ for some function f, and Kamps [42] considered the case for which $e^Z = 1 - e^{-\lambda X}$, $\lambda > 0$.

G. *Proportional Hazards Premium Principle*: $H[X] = \int_0^\infty [S_X(t)]^c \, dt$, for some $0 < c < 1$.

The Proportional Hazards Premium Principle is a special case of Wang's Premium Principle, Principle I below. Wang [3] studied the many nice properties of this premium principle.

H. *Principle of Equivalent Utility*: $H[X]$ solves the equation

$$u(w) = E[u(w - X + H)] \qquad (1)$$

where u is an increasing, concave utility of wealth (of the insurer), and w is the initial wealth (of the insurer).

One can think of H as the minimum premium that the insurer is willing to accept in exchange for insuring the risk X. On the left-hand side of (1), we have the utility of the insurer who does not accept the insurance risk. On the right-hand side, we have the expected utility of the insurer who accepts the insurance risk for a premium of H. $H[X]$ is such that the insurer is *indifferent* between not accepting and accepting the insurance risk. Thus, this premium is called the *indifference price* of the insurer. Economists also refer to this price as the *reservation price* of the insurer. The axiomatic foundations of expected utility theory are presented in [43]. See [44, 45] for early actuarial references of this economic idea and [46] for a more recent reference.

Pratt [47] studied how reservation prices change as one's **risk aversion** changes, as embodied by the utility function. Pratt showed that for 'small' risks, the Variance Premium Principle, Principle C, approximates the premium from the principle of equivalent utility with $\alpha = -(1/2)(u''(w)/u'(w))$, that is, the loading factor equals one-half the measure of absolute risk aversion. If $u(w) = -e^{-\alpha w}$, for some $\alpha > 0$, then we have the Exponential Premium Principle, Principle E.

Alternatively, if u and w represent the utility function and wealth of a buyer of insurance, then the maximum premium that the buyer is willing to pay for coverage is the solution G of the equation $E[u(w - X)] = u(w - G)$. The resulting premium $G[X]$ is the indifference price for the buyer

of insurance – she is indifferent between not buying and buying insurance at the premium $G[X]$.

Finally, the terminology of *principle of zero-utility* arises when one defines the premium F as the solution to $v(0) = E[v(F - X)]$ [24, 48]. One can think of this as a modification of (1) by setting $v(x) = u(w + x)$, where w is the wealth of the insurer. All three methods result in the same premium principle when we use exponential utility with the same value for the risk parameter α. However, one generally expects that the insurer's value for α to be less than the buyer's value. In this case, the minimum premium that the insurer is willing to accept would be less than the maximum premium that the buyer is willing to pay.

I. *Wang's Premium Principle*: $H[X] = \int_0^\infty g[S_X(t)] \, dt$, where g is an increasing, concave function that maps $[0, 1]$ onto $[0, 1]$. The function g is called a *distortion* and $g[S_X(t)]$ is called a *distorted (tail) probability*.

The Proportional Hazards Premium Principle, Principle G, is a special case of Wang's Premium Principle with the distortion g given by $g(p) = p^c$ for $0 < c < 1$. Other distortions have been studied; see [11] for a catalog of some distortions. Also, a recent addition to this list is the so-called *Wang transform* given by $g(p) = \Phi[\Phi^{-1}(p) + \lambda]$, where Φ is the cumulative distribution function for the standard normal random variable and λ is a real parameter [49, 50]. See [51] for the optimal insurance when prices are calculated using Wang's Premium Principle, and see [52–54] for further study of Wang's Premium Principle.

Wang's Premium Principle is related to the idea of coherent risk measures [55, 56], and is founded in Yaari's dual theory of risk [5]. It is also connected with the subject of nonadditive measure theory [57]. One can combine the Principle of Equivalent Utility and Wang's Premium Principle to obtain premiums based on *anticipated utility* [58–60]. Young [61] showed that Wang's Premium Principle reduces to a standard deviation for location-scale families, and Wang [62] generalized her result. Finally, Landsman and Sherris [63] offered an alternative premium principle to Wang's.

J. *Swiss Premium Principle*: The premium H solves the equation

$$E[u(X - pH)] = u((1 - p)H) \tag{2}$$

for some $p \in [0, 1]$ and some increasing, convex function u.

Note that the Swiss Premium Principle is a generalization of the principle of zero utility as defined in the discussion of the Principle of Equivalent Utility, Principle H. The Swiss Premium Principle was introduced by Bühlmann et al. [64]. This premium principle was further studied in [65–68].

K. *Dutch Premium Principle*: $H[X] = EX + \theta E[(X - \alpha EX)_+]$, with $\alpha \geq 1$ and $0 < \theta \leq 1$.

Van Heerwaarden and Kaas [69] introduced this premium principle, and Hürlimann [70] extended it to experience rating and reinsurance.

As shown in the table on the following page, we catalog which properties from the section 'Properties of Premium Principles' are satisfied by the premium principles listed above. A 'Y' indicates that the premium principle satisfies the given property. An 'N' indicates that the premium principle does not satisfy the given property for all cases.

The Characterization Method

In this section, we demonstrate the characterization method. First, we provide a theorem that shows how Wang's Premium Principle can be derived from this method. See [4] for discussion and more details.

Theorem 1 *If the premium principle $H : \chi \to \overline{\mathbf{R}}^+ = [0, \infty]$, satisfies the properties of Independence (Property 1), Monotonicity (Property 12), Comonotonic Additivity (Property 11), No Unjustified Risk Loading (Property 3), and Continuity (Property 15), then there exists a nondecreasing function $g : [0, 1] \to [0, 1]$ such that $g(0) = 0$, $g(1) = 1$, and*

$$H[X] = \int_0^\infty g[S_X(t)] \, dt \tag{3}$$

Furthermore, if χ contains all the Bernoulli random variables, then g is unique. In this case, for $p \in [0, 1]$, $g(p)$ is given by the price of a Bernoulli(p) (see **Discrete Parametric Distributions**) *risk.*

Note that the result of the theorem does not require that g be concave. If we impose the property of Subadditivity, then g will be concave, and we have Wang's Premium Principle.

Next, we obtain the Proportional Hazards Premium Principle by adding another property to those in

Premium principle Letter		A	B	C	D	E	F	G	H	I	J	K
		Net	Exp'd value	Var	Std dev	Exp	Esscher	PH	Equiv utility	Wang	Swiss	Dutch
Property Number	Name											
1	Independent	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
2	Risk load	Y	Y	Y	Y	Y	N (Y if $Z = X$)	Y	Y	Y	Y	Y
3	Not unjustified	Y	N	Y	Y	Y	Y	Y	Y	Y	Y	Y
4	Max loss	Y	N	N	N	Y	Y	Y	Y	Y	Y	Y
5	Translation	Y	N	Y	Y	Y	Y	Y	Y	Y	N	N
6	Scale	Y	Y	N	Y	N	N	Y	N	Y	N	Y
7	Additivity	Y	Y	N	N	N	N	N	N	N	N	N
8	Subadditivity	Y	Y	N	N	N	N	Y	N	Y	N	Y
9	Superadditivity	Y	Y	N	N	N	N	N	N	N	N	N
10	Add indep.	Y	Y	Y	N	Y	N	N	N	N	N	N
11	Add comono.	Y	Y	N	N	N	N	Y	N	Y	N	N
12	Monotone	Y	Y	N	N	Y	N	Y	Y	Y	Y	Y
13	FSD	Y	Y	N	N	Y	N	Y	Y	Y	Y	Y
14	SL	Y	Y	N	N	Y	N	Y	Y	Y	Y	Y
15	Continuity	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y

Theorem 1. See [4] for a discussion of this additional property from the viewpoint of no-arbitrage in financial markets.

Corollary 1 *Suppose H is as assumed in Theorem 1 and that H additionally satisfies the following property:*

Let X = I Y be a compound Bernoulli random variable, with X, I, and Y $\in \chi$, where the Bernoulli random variable I is independent of the random variable Y. Then, H[X] = H[I]H[Y].

It follows that there exists c > 0, such that $g(p) = p^c$, for all $p \in [0, 1]$.

Finally, if we assume the property of Risk Loading (Property 2), then we can say that c in Corollary 1 lies between 0 and 1. Thus, we have characterized the Proportional Hazards Premium Principle as given in the section, 'Catalog of Premium Principles: The Ad Hoc Method'.

Other premium principles can be obtained through the characterization method. For example, Promislow [71, 72] and Promislow and Young [73–75] derived premium principles that are characterized by properties that embody the economic notion of *equity*. This observation leads us nicely into the last method of obtaining premium principles, namely, the economic method.

The Economic Method

In this section, we demonstrate the economic method for deriving premium principles. Perhaps the most well-known such derivation is in using expected utility theory to derive the Principle of Equivalent Utility, Principle H. Similarly, if we use Yaari's dual theory of risk in place of expected utility theory, then we obtain Wang's Premium Principle. See our discussion in the section, 'Catalog of Premium Principles: The Ad Hoc Method' where we first introduce these two premium principles.

The Esscher Premium Principle can be derived from expected utility theory as applied to the problem of risk exchanges [1, 2, 44]. Suppose that insurer j faces risk X_j, $j = 1, 2, \ldots, n$. Assume that insurer j has an exponential utility function given by $u_j(w) = 1 - \exp(-\alpha_j w)$. Bühlmann [1, 2] defined an equilibrium to be such that each agent's expected utility is maximized. This equilibrium also coincides with a **Pareto optimal** exchange [35, 36, 44, 45].

Theorem 2 *In equilibrium, the price for risk X is given by*

$$H[X] = \frac{E[Xe^{\alpha Z}]}{E[e^{\alpha Z}]} \qquad (4)$$

in which $Z = X_1 + X_2 + \cdots + X_n$ is the aggregate risk, and $(1/\alpha) = (1/\alpha_1) + (1/\alpha_2) + \cdots + (1/\alpha_n)$.

Theorem 2 tells us that in a Pareto optimal risk exchange with exponential utility, the price is given by the Esscher Premium Principle with Z equal to the aggregate risk and $1/\alpha$ equal to the aggregate 'risk tolerance'. Bühlmann [2] generalized this result for insurers with other utility functions. Wang [50] compared his premium principle with the one given in Theorem 2. Iwaki, Kijima, and Morimoto [76] extended Bühlmann's result to the case for which we have a dynamic financial market in addition to the insurance market.

Summary

We have proposed three methods of premium calculation: (1) the *ad hoc method*, (2) the *characterization method*, and (3) the *economic method*. In the section, 'Catalog of Premium Principles: The Ad Hoc Method', we demonstrated the ad hoc method; in the section 'The Characterization Method', the characterization method for Wang's Premium Principle and the Proportional Hazards Premium Principle; and in the section 'The Economic Method', the economic method for the Principle of Equivalent Utility, Wang's Premium Principle, and the Esscher Premium Principle. For further reading about premium principles, please consult the articles and texts referenced in this paper, especially the general text [77].

Some of the premium principles mentioned in this article can be extended to dynamic markets in which either the risk is modeled by a **stochastic process**, the financial market is dynamic, or both. See [19, 78, 79] for connections between actuarial and financial pricing. For more recent work, consult [27, 28, 33], and the references therein.

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(*See also* **Equilibrium Pricing Model**; **Ordering of Risks**; **Utility Theory**)

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